The Expressiveness of Concentration Controlled P Systems

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Overview

- Introduction
- Concentration controlled P Systems
- Results

Introduction to P Systems

- Introduced by Gh. Păun (Nov 1998)
- $\Pi = (V, C, \mu, w_1, \ldots, w_n, R_1, \ldots, R_n, i_0)$
- Multisets of objects w_i associated with regions i , and rules R_i to control the evolution/communication of objects
- A catalyst $(c \in C)$ is a special object which controls evolution/communication

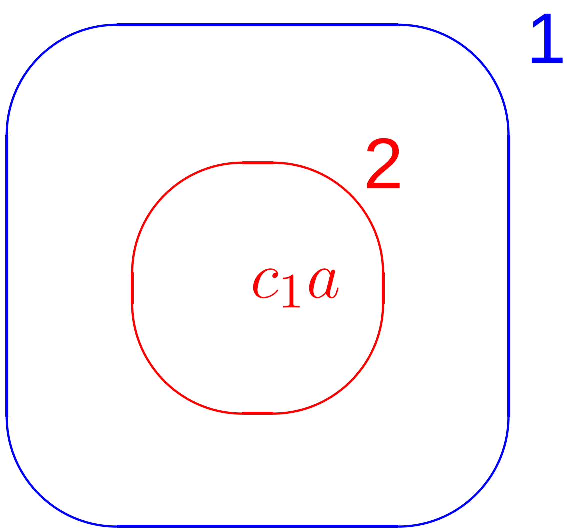
- Rules in a region are of the kind $a \rightarrow (w, tar)$, $ca \rightarrow c(w, tar)$, c catalyst, $a \in V - C$
- A computation is a sequence of configurations of the system
- Output : The number of objects in a specified output membrane at the end of a halting computation

Concentration controlled P Systems

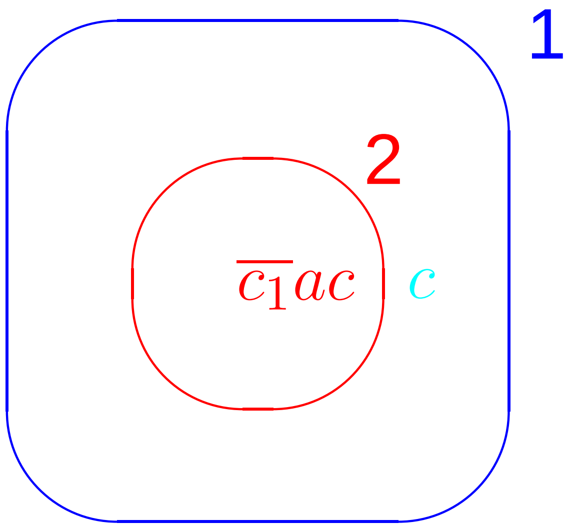
- $\Pi = (V, V_c, T, C, \mu, R_1, \ldots, R_n)$,
- $V_c \subseteq V$ objects whose concentration will be controlled,
  - objects of V_c redistributed : all regions have equal number of symbols of V_c; a difference of one allowed if number of symbols not divisible by the number of regions.
- C is a set of bi-stable catalysts, $C \cap V = \emptyset$
  - Rules of the form $ca \rightarrow \bar{c}v, \bar{c}a \rightarrow cv$.
  - Output: Number of terminal symbols outside the syetm at the end of a halting configuration.

Example

- Consider the system $(\{a, c\}, \{c\}, \{c\}, \{c_1\}, [\,[\,a\,]_2\,]_1, R_1, R_2)$ with
- $R_2 = \{c_1 a \rightarrow \overline{c_1} a c^2, \overline{c_1} c \rightarrow c_1 \lambda\}, R_1 = \{c \rightarrow c^3, c \rightarrow \lambda\}$.



Example



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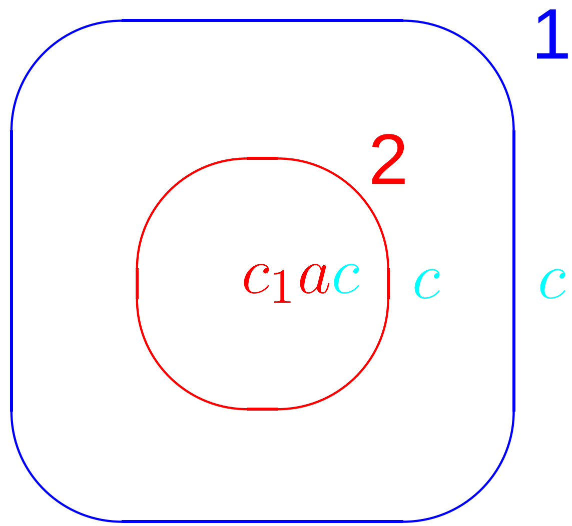

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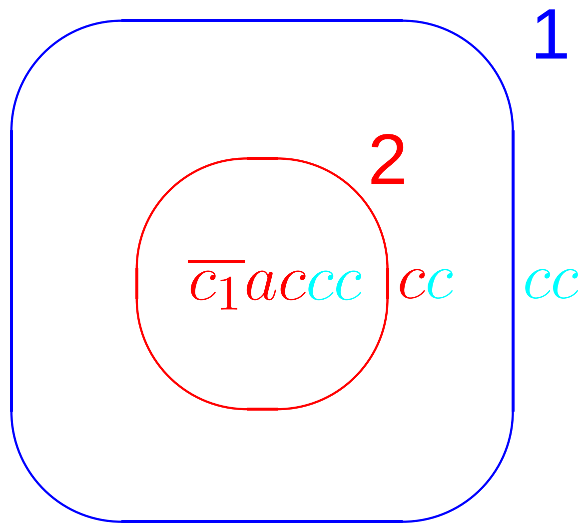


Results, Questions

- arbitrarily many bi-stable catalysts - universal (J.Dassow, Gh. Păun, 2001)
- 1 bi-stable catalyst - MAT (J.Dassow, Gh. Păun, 2001)
- Expressiveness results using catalysts, mobile catalysts, optimal results for bi-stable catalysts

Bi-stable catalysts

- 2 bi-stable catalysts, 1 membrane - universal
  - Simulation of a 2 counter machine
- 1 bi-stable catalyst, 1 membrane - MAT^{λ}
  - Construction of a random context matrix grammar without appearance checking.

Catalysts

- 1 membrane + 2 catalysts - universal
  - Simulation of a 2 counter machine
- 1 membrane + 1 catalyst - non-universal
  - Construction of a random context grammar without appearance checking

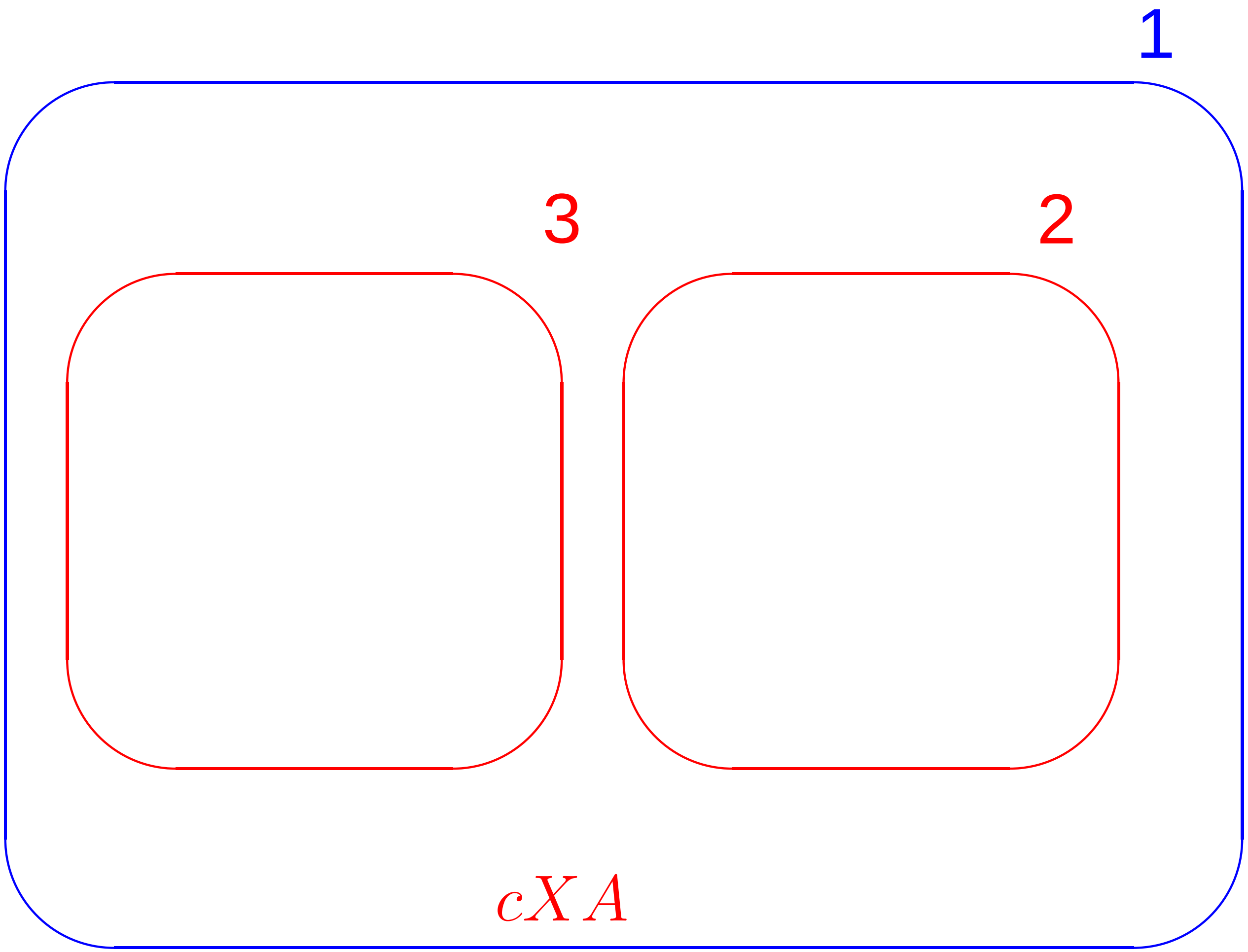
Mobile Catalysts

- Catalysts which can move to adjacent regions - rules $ca \rightarrow (c, move)w$
- 3 membranes + 1 mobile catalyst - universal
  - Simulation of matrix grammars with appearance checking
- 2 membranes + 1 mobile catalyst - non-universal
  - $\Pi \rightarrow \Pi'$ such that $\mathbb{N}(\Pi) = \mathbb{N}(\Pi')$. Π: concentration controlled, Π': P systems with explicit targets for all symbols
  - Use known result for Π'.

Universality using Mobile Catalysts

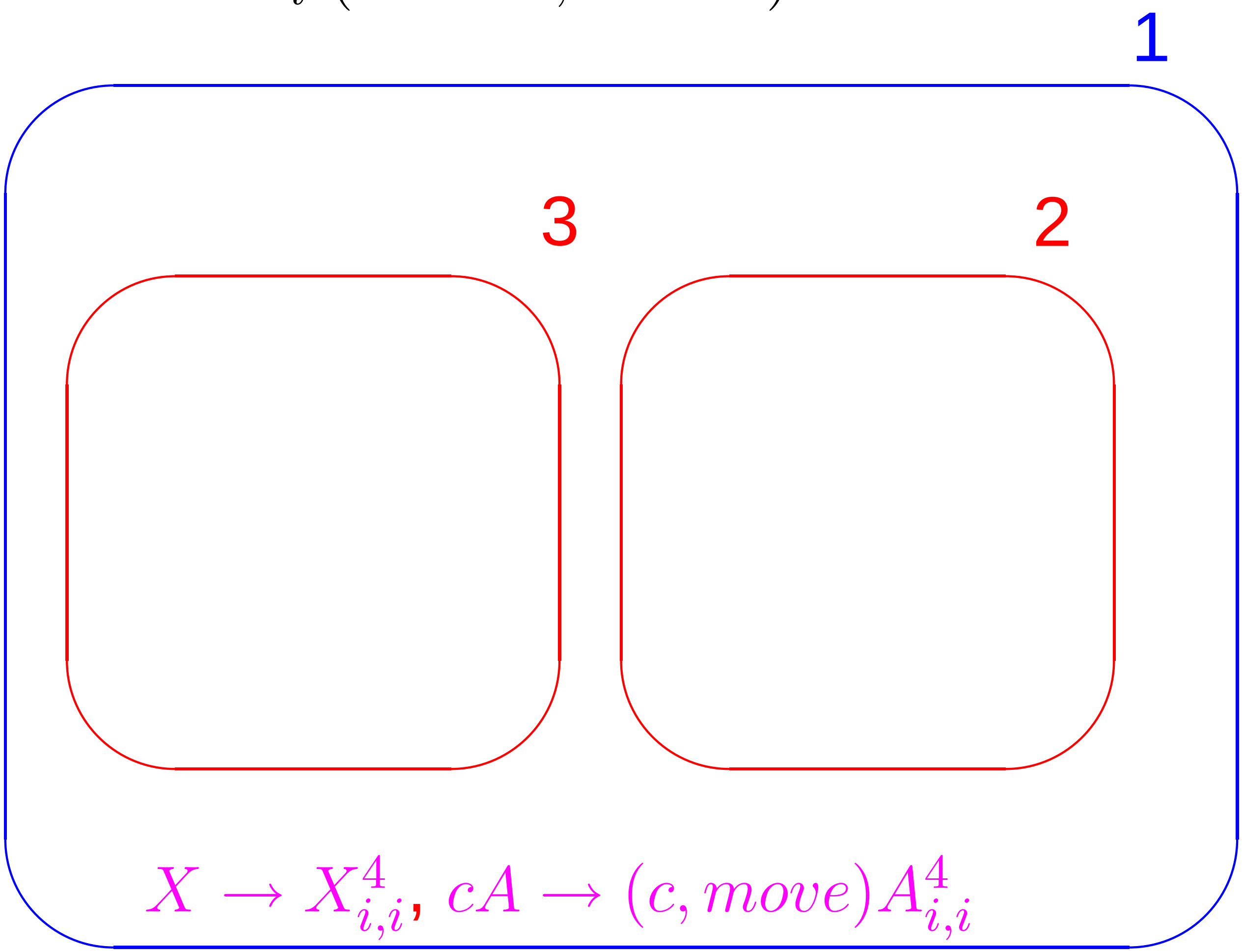
- Simulate matrix grammars with appearance checking: $G = (N, T, S, M, F)$ matrices of the form
  - $(S \rightarrow XA)$, with $X \in N_1, A \in N_2,$
  - $(X \rightarrow Y, A \rightarrow x)$, with $X, Y \in N_1, A \in N_2, x \in (N_2 \cup T)^*$, $|x| \leq 2$,
  - $(X \rightarrow Y, B^{(j)} \rightarrow \#)$, with $X, Y \in N_1, j = 1, 2,$
  - $(X \rightarrow \lambda, A \rightarrow x)$, with $X \in N_1, A \in N_2, x \in T^*$.

Universality using Mobile Catalysts



Universality using Mobile Catalysts

Simulation of m_i:$(X \to Y, A \to x)$



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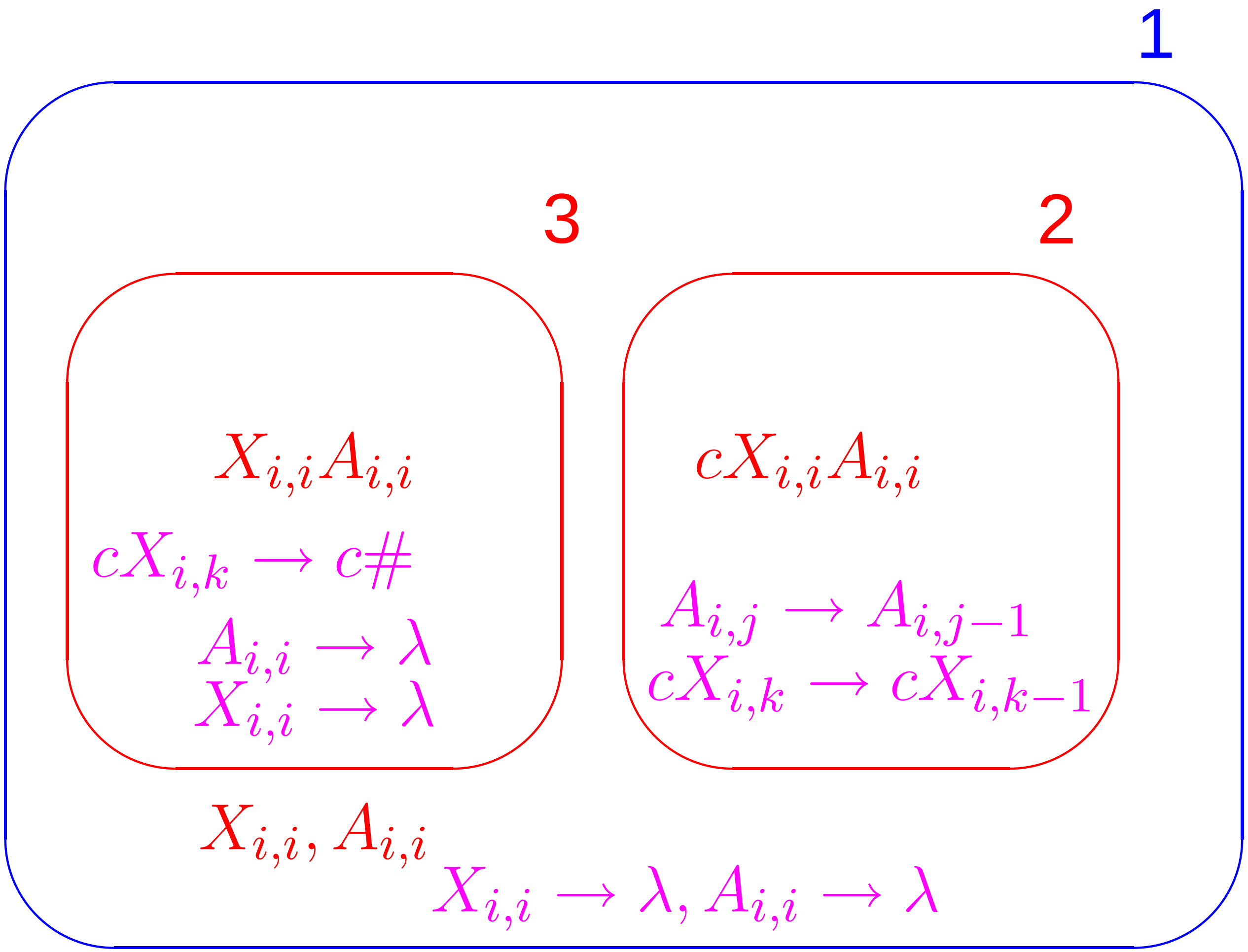
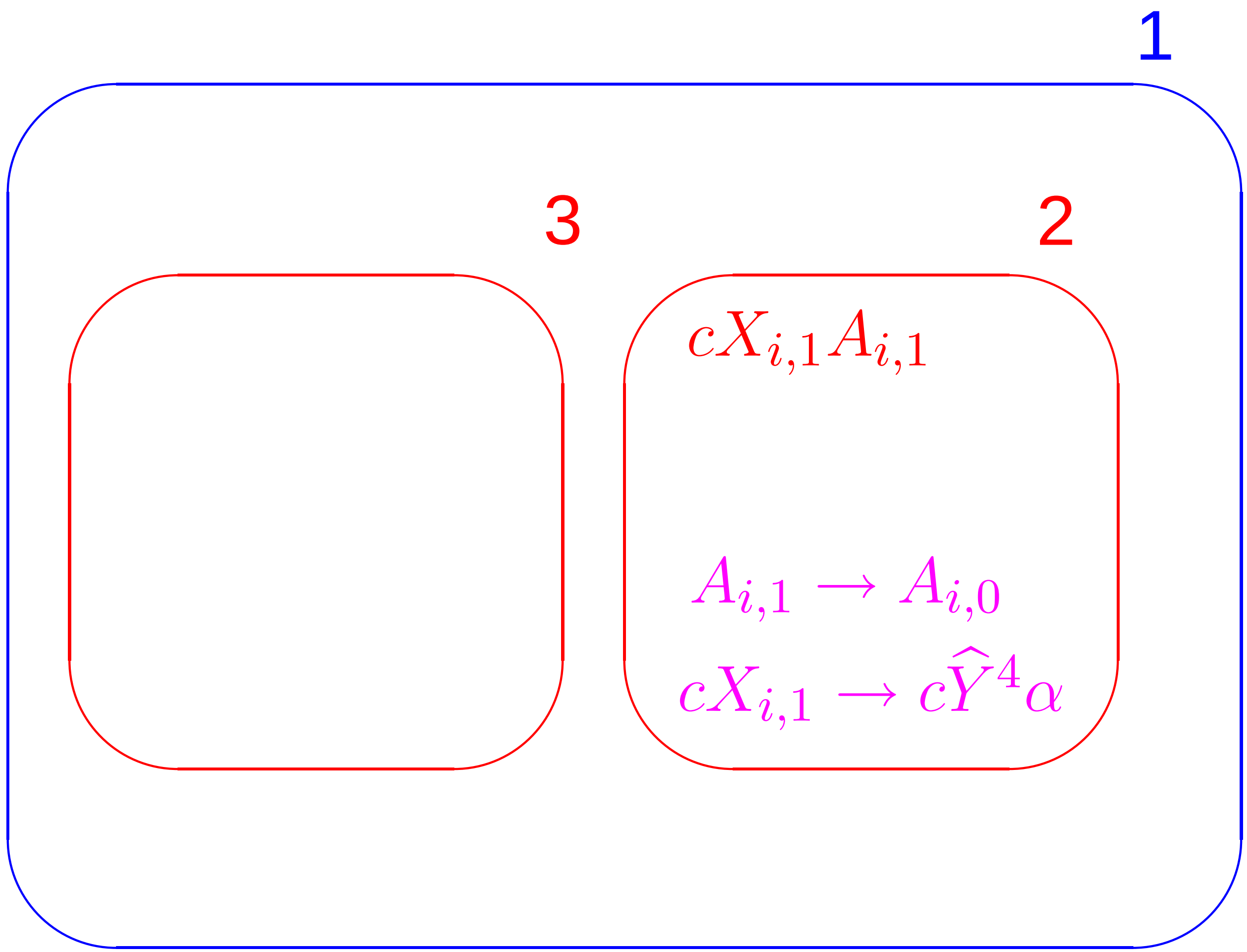


$X_{i,i}, A_{i,i}$

Universality using Mobile Catalysts



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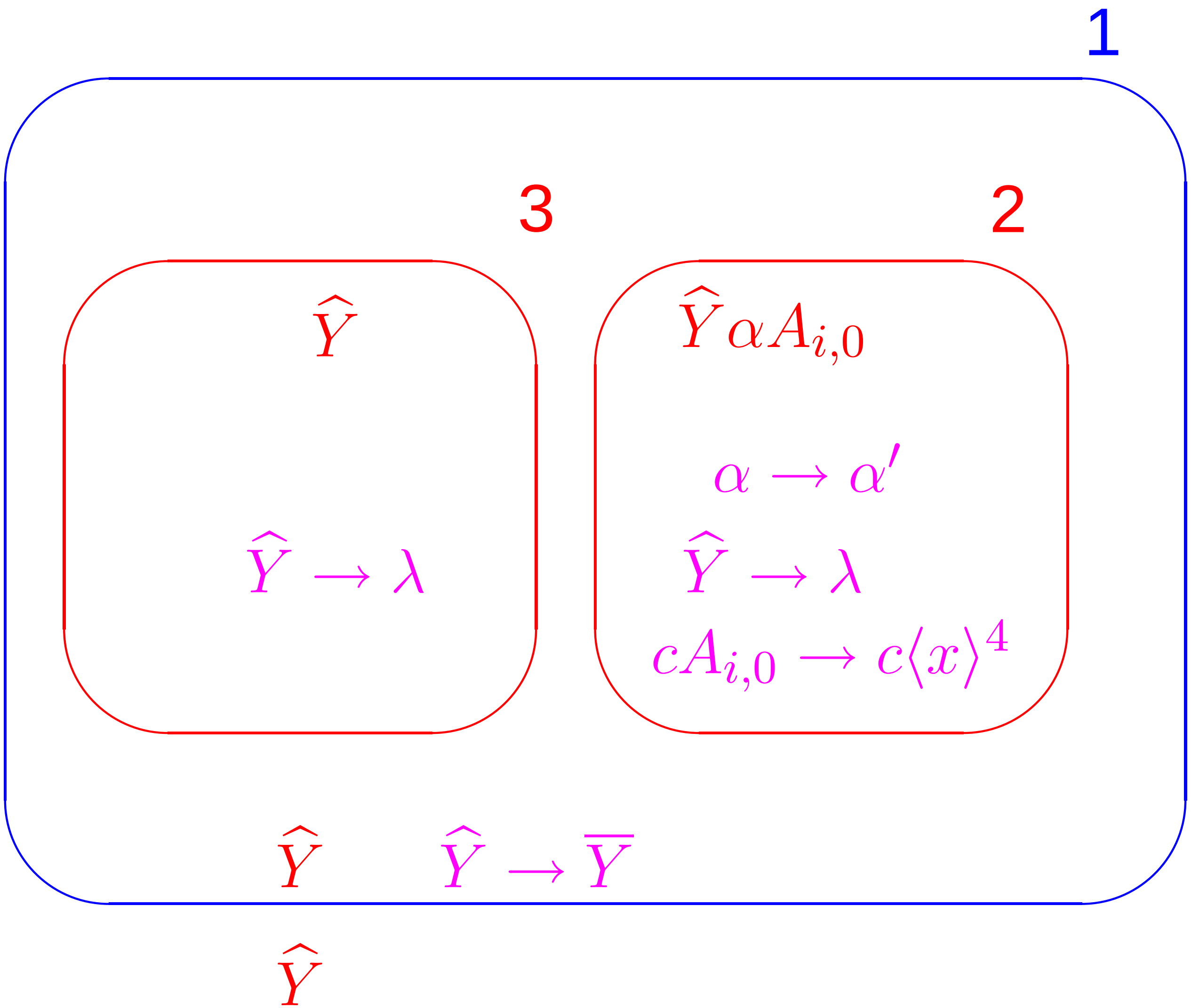

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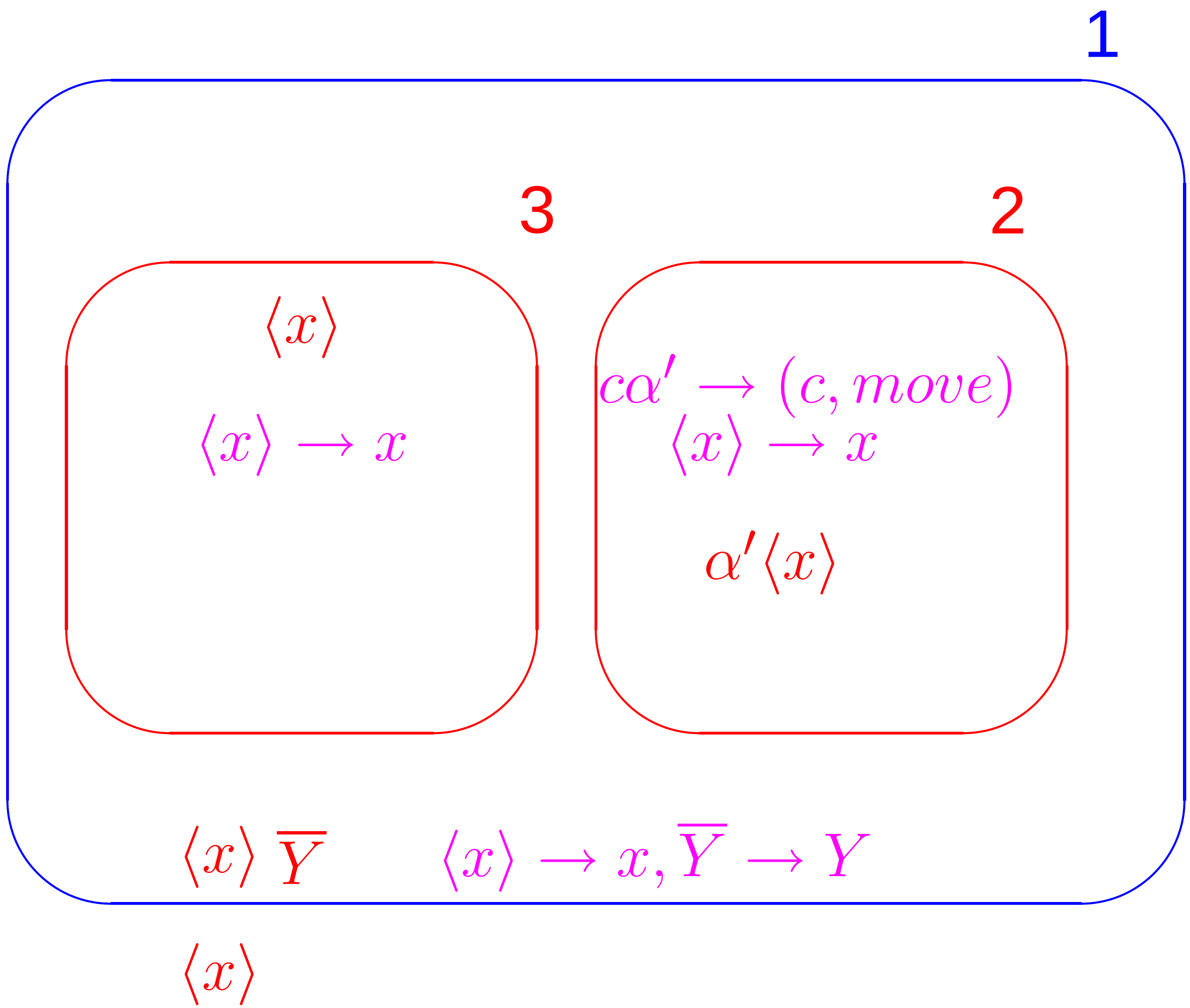

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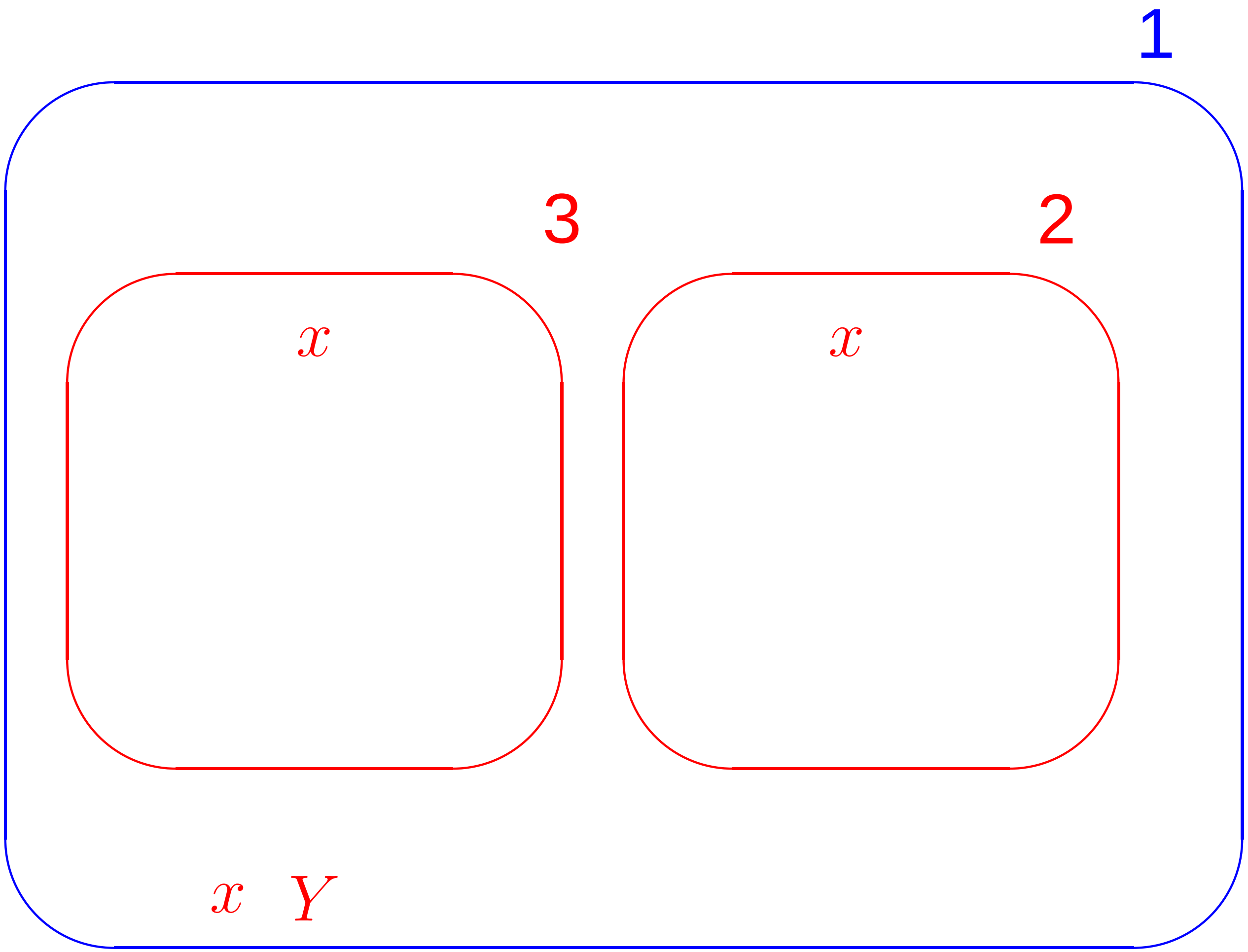
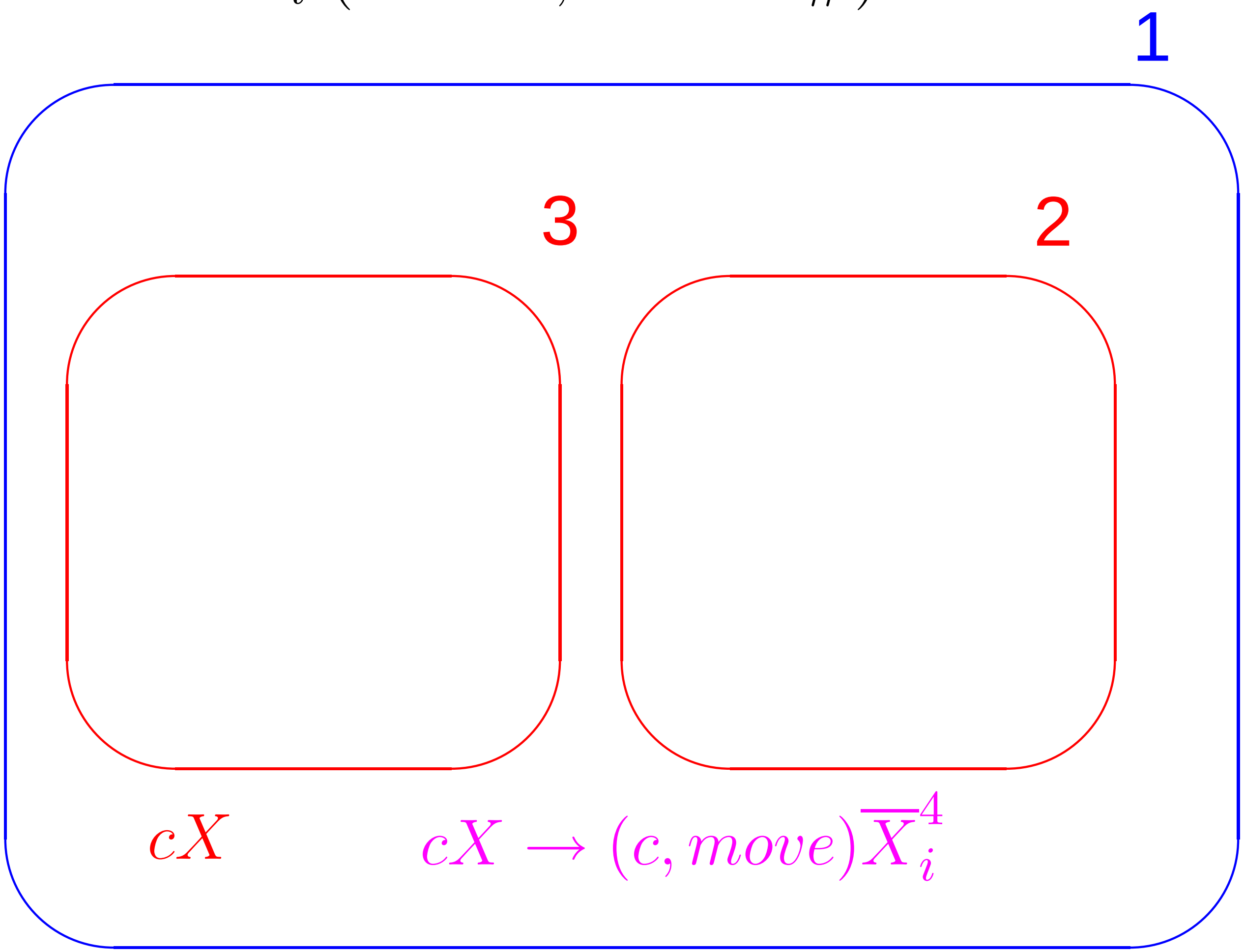


Universality using Mobile Catalysts

Simulation of m_i:$(X \rightarrow Y, B^{(1)} \rightarrow \#)$



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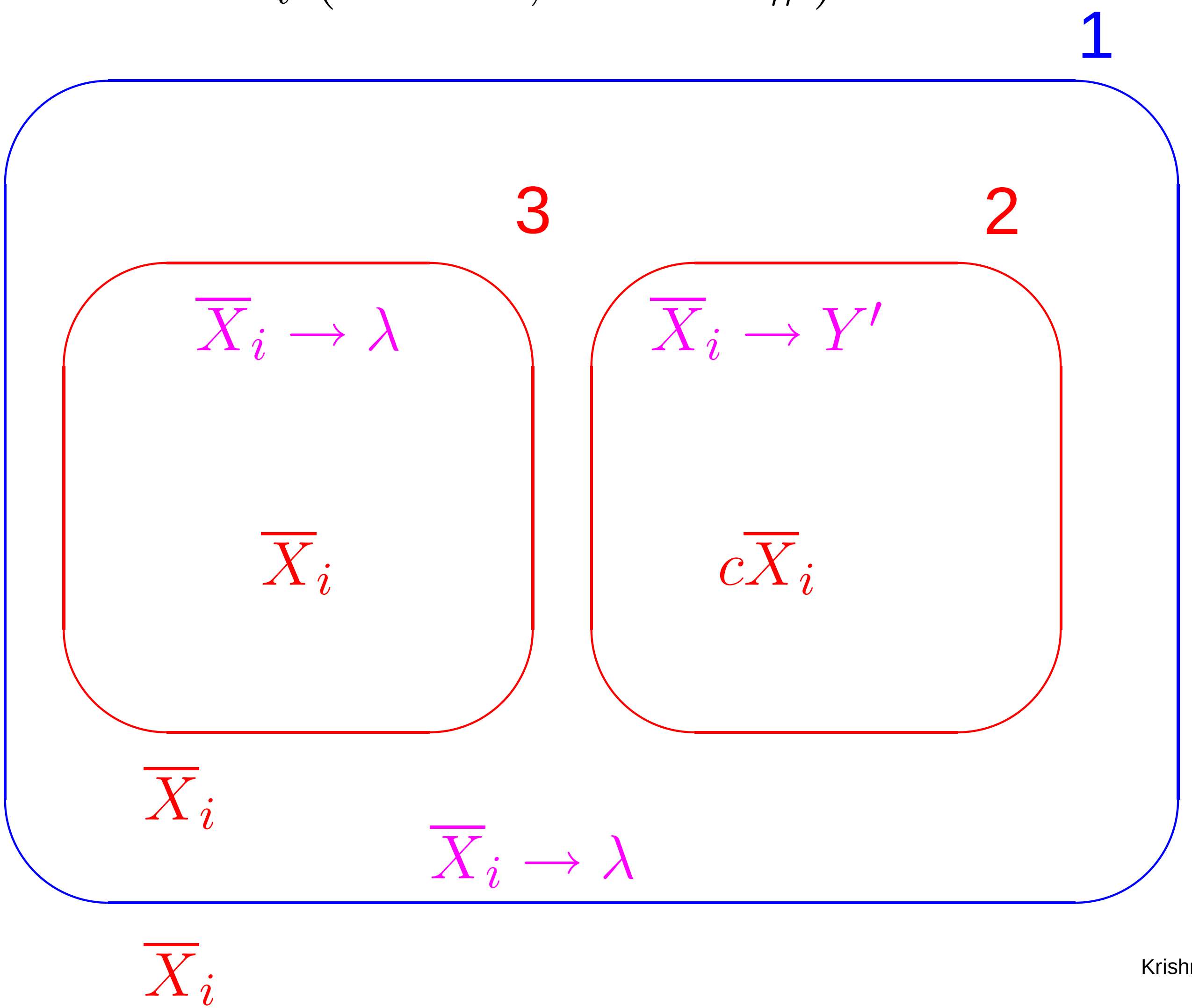

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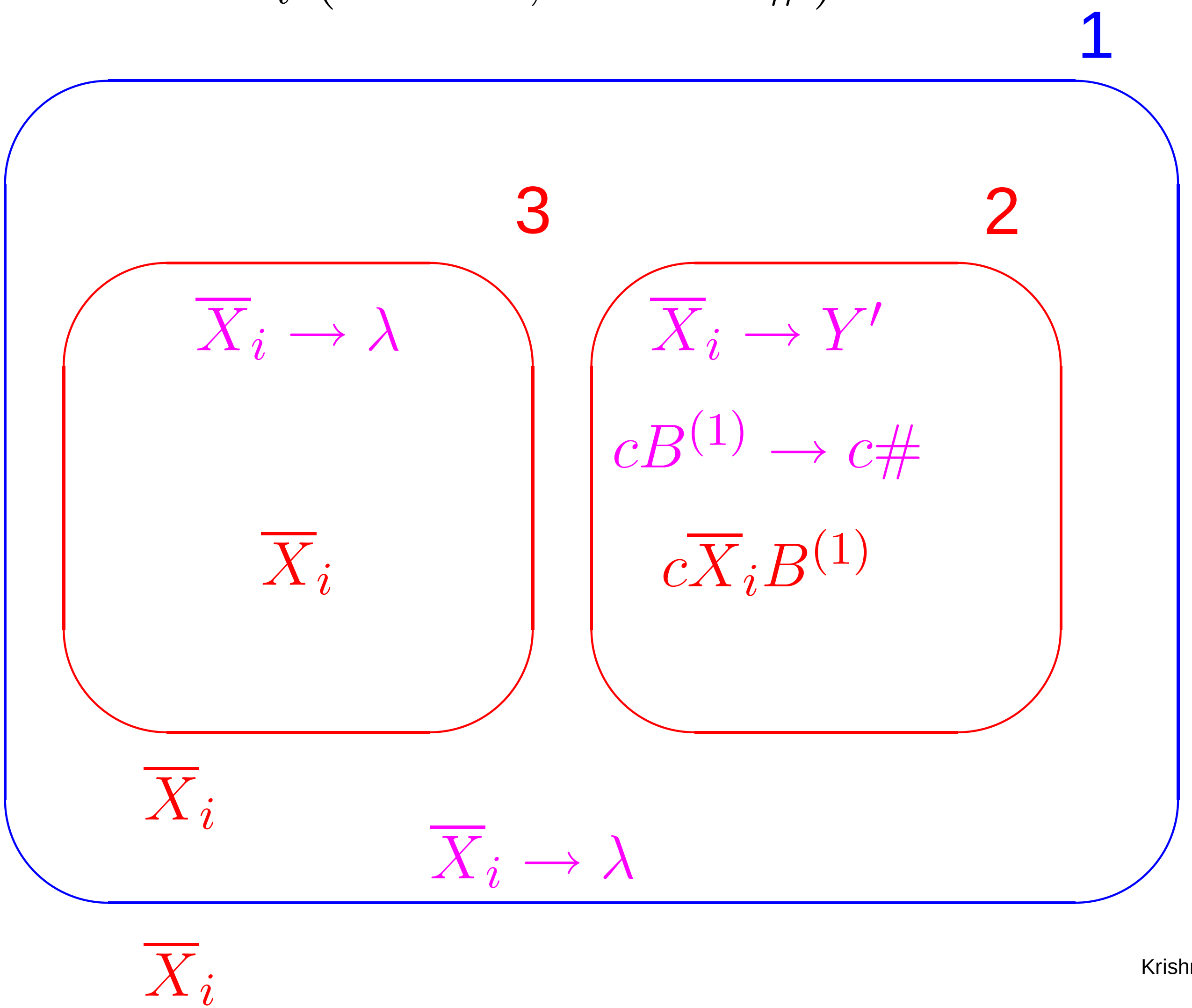

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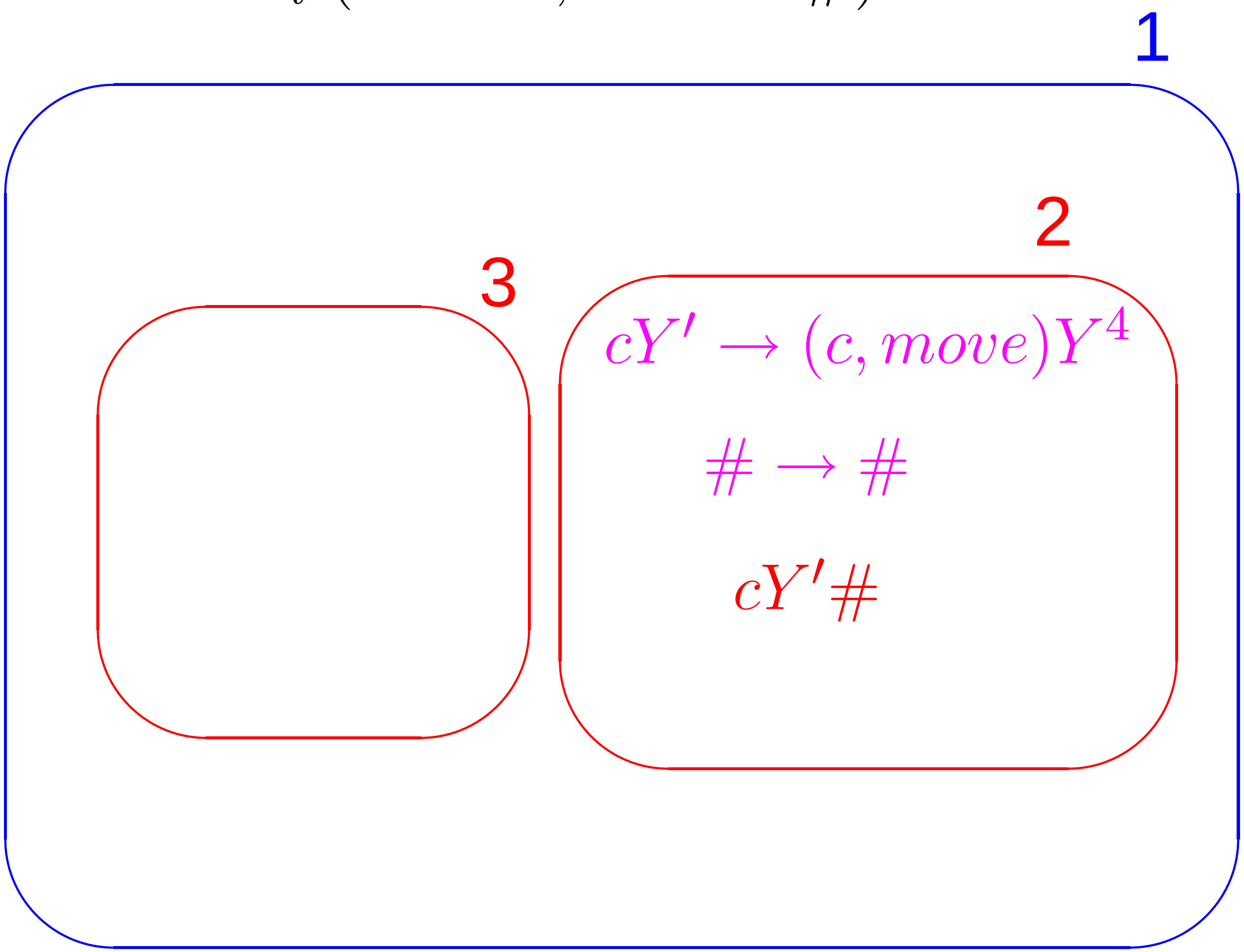

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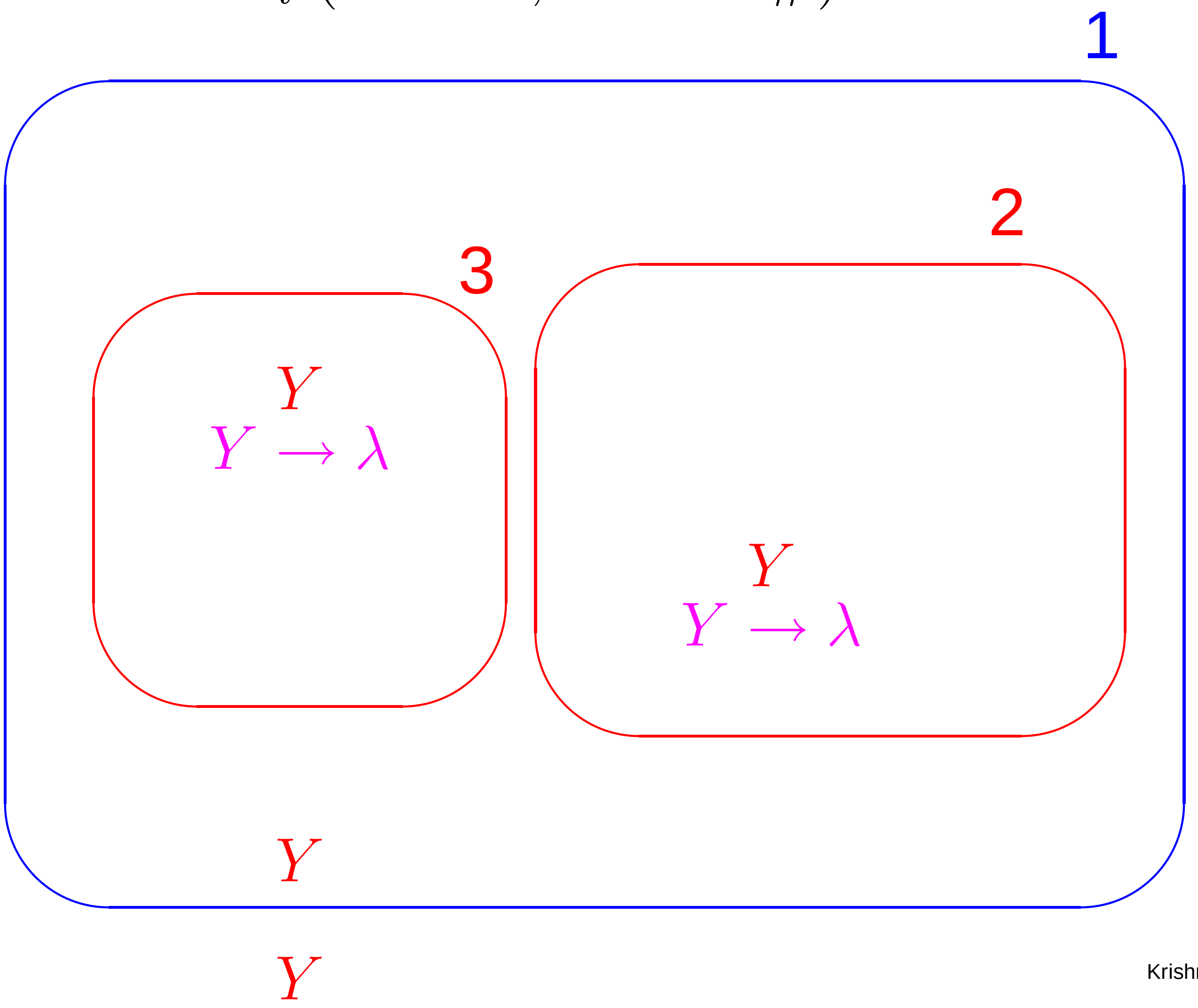

Universality using Mobile Catalysts

- The simulation of $m_i : (X \rightarrow Y, B^{(2)} \rightarrow \#)$ is similar.
- At the end of a halting computation, the terminal symbols sent out forms the output.

Conclusion

- Bi-stable catalysts - improved results
- Catalysts, mobile catalysts - new results
- Open Questions : Optimality, descriptional complexity measures, applications....