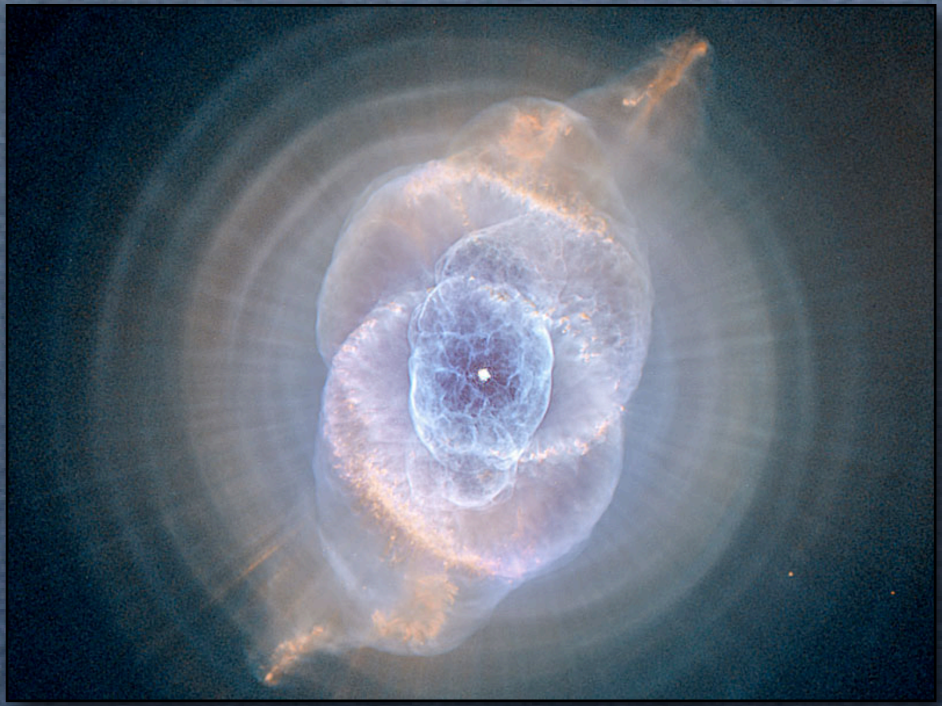
Emergence as a Computability Theoretic Phenomenon

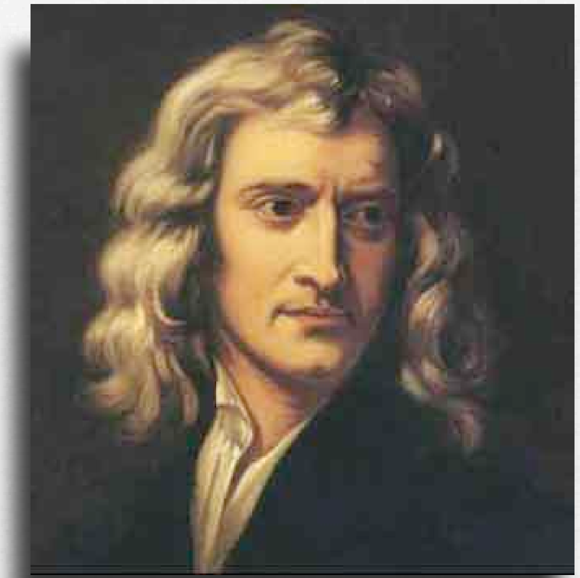


S. Barry Cooper - *Physics and Computation* International Workshop, UC 2008, Vienna, Austria, August 25 - 28, 2008

Outline:

- The Laplacian model becomes more of a model
- Some uncomfortable consequences
- What is emergence? - definability, nonlocality
- Is that all there is? - Turing and the human brain ...
- The extended Turing model, and a physics road test

The Algorithmic Content of Science

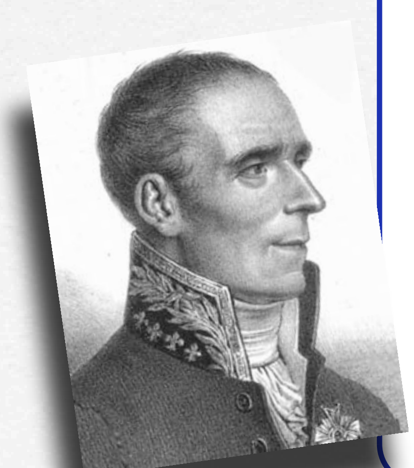


- Galileo and Newton onwards - overarching aim of science became the <u>extraction of the algorithmic content</u> of the world … theories which predict, theorems with proofs …
- Einstein [p.54, `Out of My Later Years', 1950]: "When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them."

Laplace's Predictive 'Demon' as model

"Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situations of the beings who compose it - an intelligence sufficiently vast to submit these data to analysis - it would embrace in the same formula the movements of the greatest bodies and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes."

from P. S. de Laplace [1819], "Essai philosophique sur les probabilités"



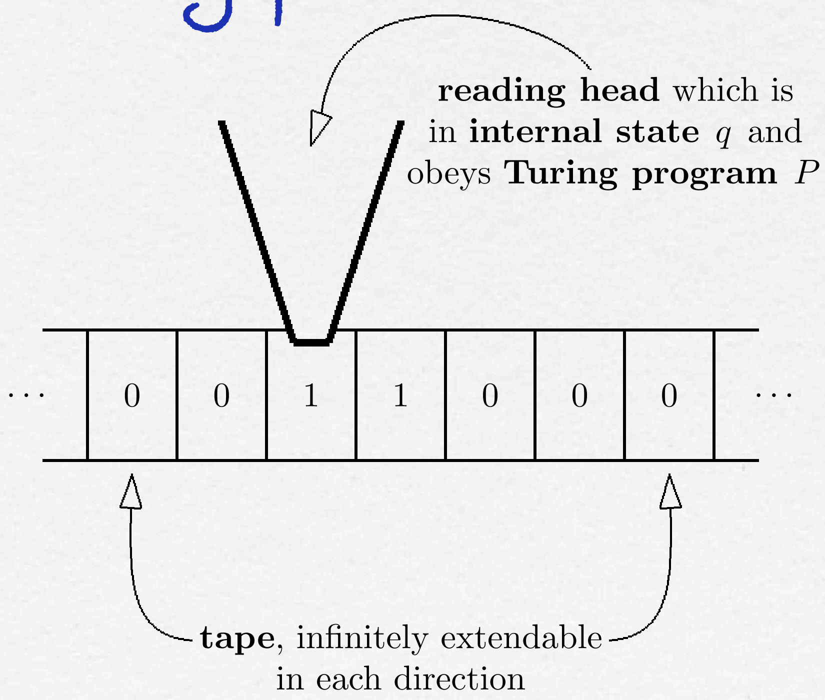
Hilbert's Programme

"For the mathematician there is no Ignorabimus, and, in my opinion, not at all for natural science either. ... The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem.
In contrast to the foolish Ignorabimus, our credo avers:
We must know,
We shall know. "

- David Hilbert's opening address to the Society of German Scientists and Physicians, Königsberg, September 1930

A mathematical model at last

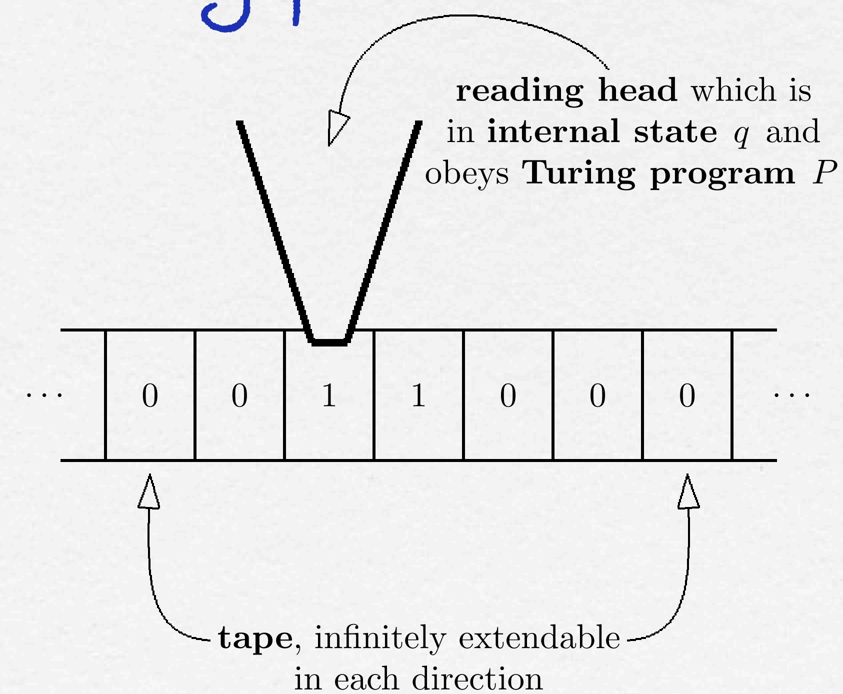
- 1936 - Turing's machines appear
- Provide a model of algorithmic natural processes within structures which are countably presented



A mathematical model at last

- 1936 - Turing's machines appear
- Provide a model of algorithmic natural processes within structures which are countably presented

But - techniques for presenting machines give the <u>universal Turing machine</u> - and <u>incomputable objects</u>



New algorithmic content …

- Incomputable computably enumerable sets
- Approximations to Δ_2 and Σ_2 sets

> “. . . if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that. "
>
> A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted by Andrew Hodges in “Alan Turing - the enigma”, p.361

Natural phenomena as discipline problem

- Successful reduction of "natural" examples to the Turing model - e.g. quantum computation (David Deutsch)

> I am sure
> we will have [conscious computers], I expect they will be purely classical, and I expect that it will be a long time in the future. Significant advances in our philosophical understanding of what consciousness is, will be needed.

Question and Answers with David Deutsch, on New.Scientist.com News Service, December, 2006

Natural phenomena as discipline problem

- Martin Davis versus the hypercomputationalists (Jack Copeland et al) -

> The great success of modern computers as all-purpose algorithm-executing engines embodying Turing's universal computer in physical form, makes it extremely plausible that the abstract theory of computability gives the correct answer to the question 'What is a computation?', and, by itself, makes the existence of any more general form of computation extremely doubtful.

Martin Davis [2004], The myth of hypercomputation. In Alan Turing: Life and legacy of a great thinker (C. Teuscher, ed.), Springer-Verlag

But back in the real world ...

- Persistence of problems of predictability - quantum uncertainty, emergent phenomena, chaos and strange attractors, relativity and singularities (black holes)

- Renewed interest in analog and hybrid computing machines leading to: "... the classical Turing paradigm may no longer be fully appropriate to capture all features of present-day computing."

- J. van Leeuwen, J. Wiedermann, The Turing Machine Paradigm in Contemporary Computing. In Mathematics Unlimited - 2001 and Beyond, LNCS, 2000

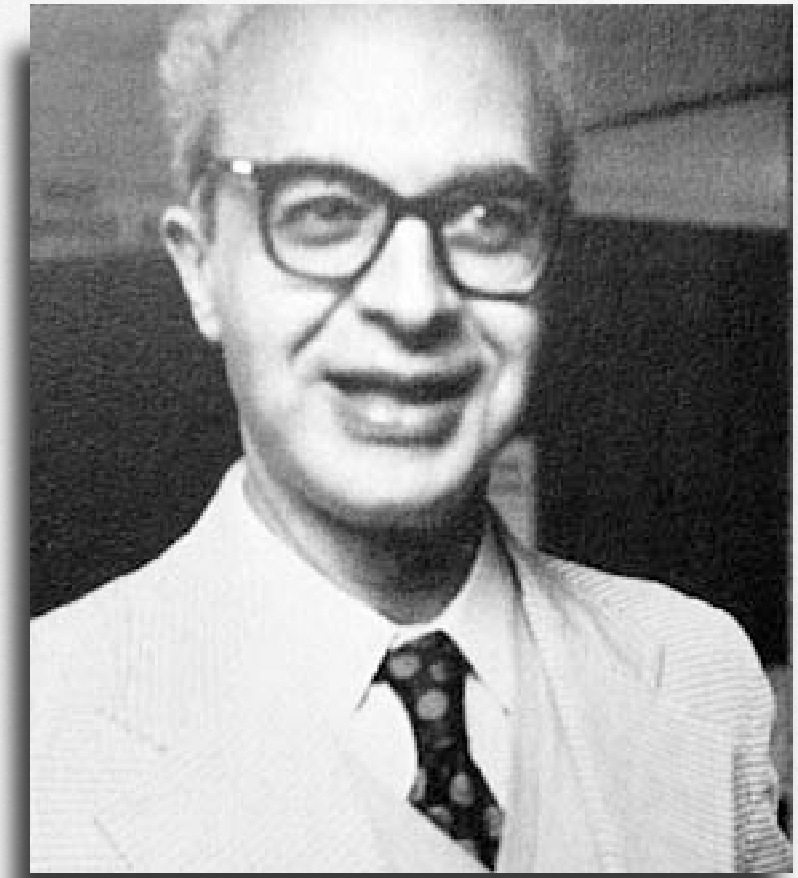
"Von Neumann's axioms distinguished the **U** (unitary evolution) and **R** (reduction) rules of quantum mechanics. Now, quantum computing so far (in the work of Feynman, Deutsch, Shor, etc) is based on the **U** process and so computable. It has not made serious use of the **R** process: the unpredictable element that comes in with reduction, measurement, or collapse of the wave function."

Andrew Hodges

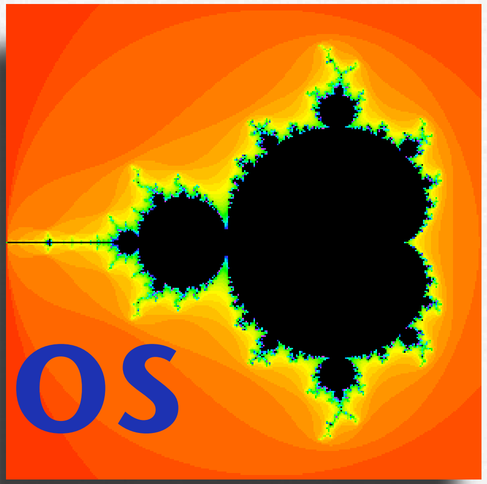
in "What would Alan Turing have done after 1954?", from Teuscher, "Alan Turing: Life and legacy of a great thinker"

Co-operative phenomena

1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in "an analog computation of a non-recursive function"



Mathematical analogues of chaos



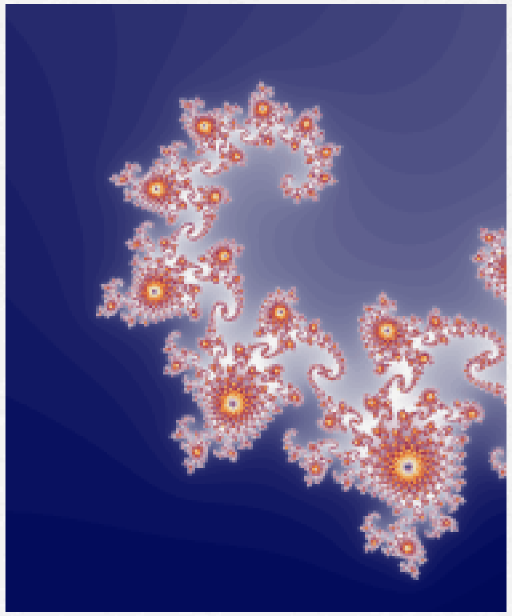
- Growth of Chaos theory, generation of informational complexity via very simple rules, accompanied by the emergence of new regularities - e.g. Robert Shaw's dripping tap[1984]
- Link between structures in nature, and mathematical objects, such as the Mandelbrot and Julia sets
- Penrose, Smale - computability of Mandelbrot, Julia sets?

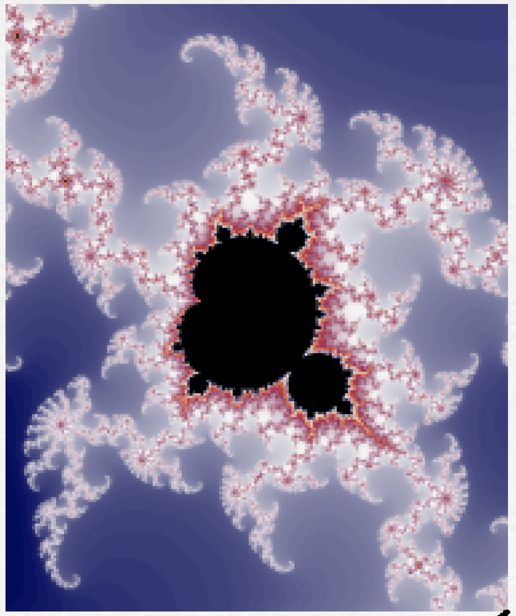
Now we witnessed ... a certain extraordinarily complicated looking set, namely the Mandelbrot set. Although the rules which provide its definition are surprisingly simple, the set itself exhibits an endless variety of highly elaborate structures.

Roger Penrose

in "The Emperor's New mind", Oxford Univ. Press, 1994

Recent results - Braverman [1999], Hertling [2005], Rettinger [2005], Rettinger and Weihrauch [2003]





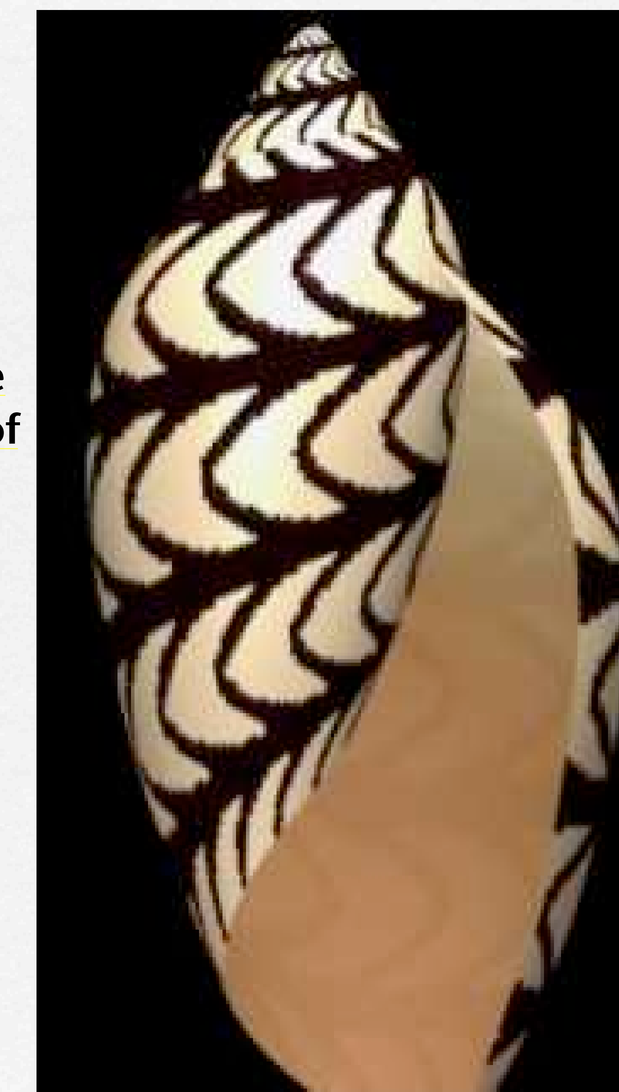
Emergence occurs everywhere ...

Cat's Eye Nebula

Emergence of patterns in Nature

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis, i.e., the development of form and shape in biological systems.

From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary:



See http://www.swintons.net/jonathan/turing.htm

Big Claims -

Emergence is often invoked in an almost mystical sense regarding the capabilities of behavior-based systems. Emergent behavior implies a holistic capability where the sum is considerably greater than its parts. It is true that what occurs in a behavior-based system is often a surprise to the system's designer, but does the surprise come because of a shortcoming of the analysis of the constituent behavioral building blocks and their coordination, or because of something else?

Ronald C. Arkin in "Behaviour-Based Robotics", MIT Press, 1998, p.105

A Test for Emergence

1) **Design:** The system has been constructed by the designer, by describing local elementary interactions between components (e.g., artificial creatures and elements of the environment) in a language $\mathcal{L}_1$.

2) **Observation:** The observer is fully aware of the design, but describes global behaviors and properties of the running system, over a period of time, using a language $\mathcal{L}_2$.

3) **Surprise:** The language of design $\mathcal{L}_1$ and the language of observation $\mathcal{L}_2$ are distinct, and the causal link between the elementary interactions programmed in $\mathcal{L}_1$ and the behaviors observed in $\mathcal{L}_2$ is non-obvious to the observer - who therefore experiences surprise. In other words, there is a cognitive dissonance between the observer's mental image of the system's design stated in $\mathcal{L}_1$ and his contemporaneous observation of the system's behavior stated in $\mathcal{L}_2$.

Ronald, Sipper and Capcarrère in "Design, observation, surprise! A test of emergence", Artificial Life, 5 (1999), 225-239

Descriptions and Emergent Structure

- Notice - It is often possible to get descriptions of emergent properties in terms of the elementary actions
- E.g., this is what Turing did for the role of Fibonacci numbers in relation to the sunflower etc.
- In mathematics, it is well-known that complicated descriptions may take us beyond what is computable
- A potential source of surprise in emergence ...

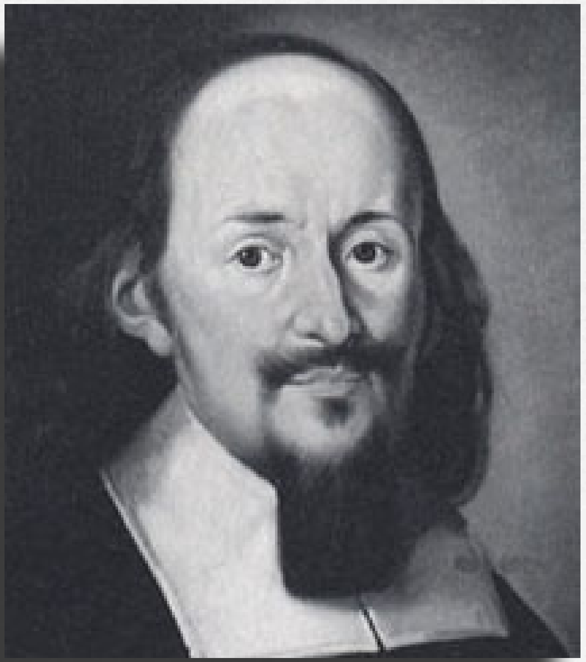
The Halting Problem Revisited

- Turing machines have simple designs, using very basic language, and are clearly observable
- Expanding the language by the addition of set theoretical notation and existential quantifiers ...
- ... get an emergent halting set which can surprise us by being incomputable
- The Mandelbrot set similarly passes the Emergence Test

Descriptions and Emergent Structure …

- Intuition - entities exist because of, and according to, mathematical laws. In the words of Leibniz [1714] -

- 'The Monadology', sections 31, 32:
  " … there can be found no fact that is true or existent, or any true proposition, without there being a sufficient reason for its being so and not otherwise, although we cannot know these reasons in most cases."



... and definability the key concept

Alfred Tarski

- So natural phenomena not only generate descriptions, but arise and derive form from them . . .
- ... so connecting with a useful abstraction - the concept of mathematical definability ...
- ... formalising describability in a mathematical structure
- Giving precision to our experience of emergence as a potentially non-algorithmic determinant of events

... and confirmed by our experience of robustness of emergent phenomena ...

I believe the following aspects of evolution to be true, without knowing how to turn them into (respectable) research topics.

Important steps in evolution are robust. Multicellularity evolved at least ten times. There are several independent origins of eusociality. There were a number of lineages leading from primates to humans. If our ancestors had not evolved language, somebody else would have.

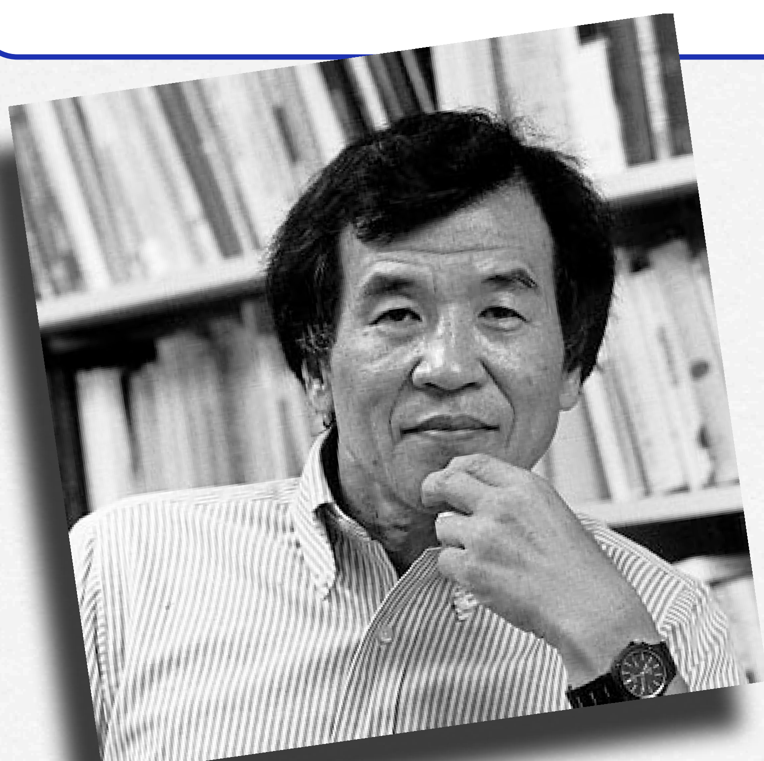
Martin Nowak,

Director, Program for Evolutionary Dynamics, Harvard University,

in John Brockman (ed.): "What We Believe But Cannot Prove"

Is the Human Mind Physical?

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."

Stanford Encyclopedia of Philosophy

Is the Human Mind Physical?

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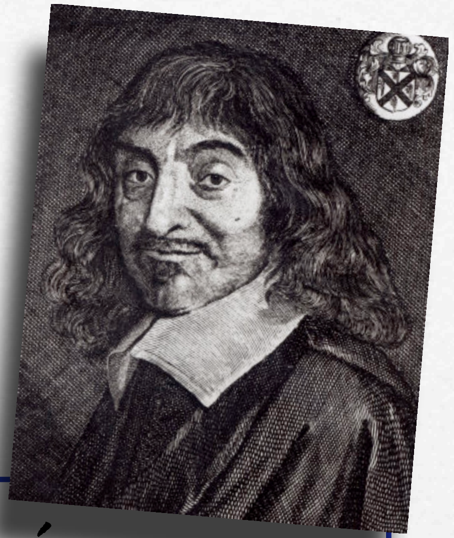
from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

- The role of a clarified notion of emergence in pinning down the nature of supervenience - and so, of intelligence
- Physicalism and consciousness reconciled ...

Descartes revisited ...



- A non-reductive physicalism, definability delivering -
- Mind-body supervenience
- The physical irreducibility of the mental - including consciousness, qualia
- And the causal efficaciousness of the mental
- With removal of conflict between 'vertical' determination and 'horizontal' causation

Emergence and Mathematical Intuition

"At first Poincaré attacked [a problem] vainly for a fortnight, attempting to prove there could not be any such function ... [quoting Poincaré]:

'Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it ... I did not verify the idea ... I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience sake, I verified the result at my leisure.' "

from Jacques Hadamard [1945], "The Psychology of Invention in the Mathematical Field", Princeton Univ. Press

Intelligent thoughts as emergent phenomena

- Need to bridge the gap between 'emergent' higher mental functionality and ... <u>what</u> algorithmic 'design'?

Intelligent machines as emergent phenomena

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- Difficult - Rodney Brooks [Nature, 2001]: "neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant."

Intelligent machines as emergent phenomena

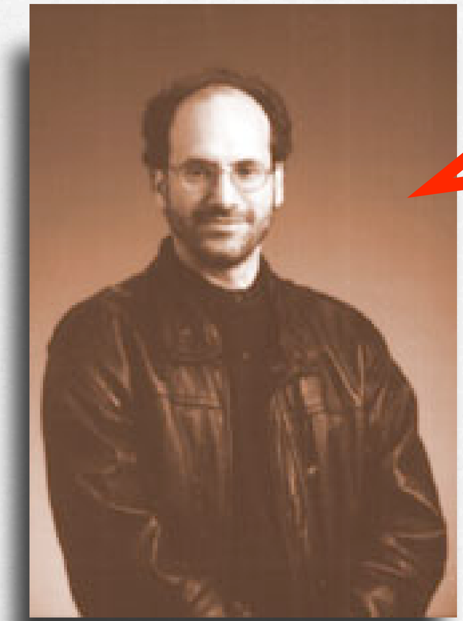
- Need to bridge the gap between 'emergent' higher mental functionality and ... what algorithmic 'design'?

- Difficult - Rodney Brooks [Nature, 2001]: "neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant."

So does emergence explain what we observe ... is that all there is ?

Connectionist Models of Computation?

There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines.



Paul Smolensky [1988] (recipient 2005 David E. Rumelhart Prize),

On the proper treatment of connectionism, in Behavioral and Brain Sciences, 11, pp. 1-74

Connectionist Models of Computation?

- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets
- But for Steven Pinker "... neural networks alone cannot do the job".

Connectionist Models of Computation?

- These have come a long way since Turing's [1948] discussion of 'unorganised machines', and McCulloch and Pitts [1943] early paper on neural nets

- But for Steven Pinker "... neural networks alone cannot do the job".

- And focussing on our elusive higher functionality, he points to a "kind of mental fecundity called recursion" . . .

We humans can take an entire proposition and give it a role in some larger proposition. Then we can take the larger proposition and embed it in a still-larger one. Not only did the baby eat the slug, but the father saw the baby eat the slug, and I wonder whether the father saw the baby eat the slug, the father knows that I wonder whether he saw the baby eat the slug, and I can guess that the father knows that I wonder whether he saw the baby eat the slug, and so on.

photo credit: www.harryborden.com

Steven Pinker,

How the Mind Works, W. W. Norton, New York, 1997

- Making a similar point - Damasio has a nice description of the hierarchical development of a particular instance of consciousness within the brain, interacting with some external object . . .

"... both organism and object are
mapped as neural patterns, in first-order maps; all of these
neural patterns can become images. ... The sensorimotor maps pertaining to the
object cause changes in the maps pertaining to the organism. ... [These] changes ... can be
re-represented in yet other maps (second-order maps) which thus represent the relationship of
object and organism. ... The neural patterns transiently formed in second-order maps
can become mental images, no less so than the neural
patterns in first-order maps."

Antonio Damasio,

The Feeling Of What Happens, Harcourt, Orlando FL, 1999

- Picture is - re-representation of neural patterns formed across some region of the brain, in such a way that they can have a computational relevance in forming new patterns

Key conception - computational loops incorporating, in a controlled way, these 'second-order' aspects of the computation itself

Turing on Description versus Computation

- Turing, 1939 - The computational content of descriptions can be captured hierarchically - but in unpredictable ways
- No consistent axiomatic theory captures arithmetic (Gödel) - but we can hierarchically transcend this barrier
- But then - identifying the route to a new theorem involves using an incomputable oracle

Despite inductive structure, reductionism breaks down

Mathematical reasoning
may be regarded ... as the exercise of a combination of
... *intuition* and *ingenuity*. ... In pre-Gödel times it was thought by some that all the intuitive judgements of mathematics could be replaced by a finite number of ... rules. The necessity for intuition would then be entirely eliminated.
In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but ingenuity, and this in spite of the fact that our aim has been in much the same direction.

Alan Turing [1939],
Systems of logic based on ordinals, Proc. London Math. Soc. (2) 45, pp.161-228.
Reprinted in A. M. Turing, Collected Works: Mathematical Logic, pp. 81-148.

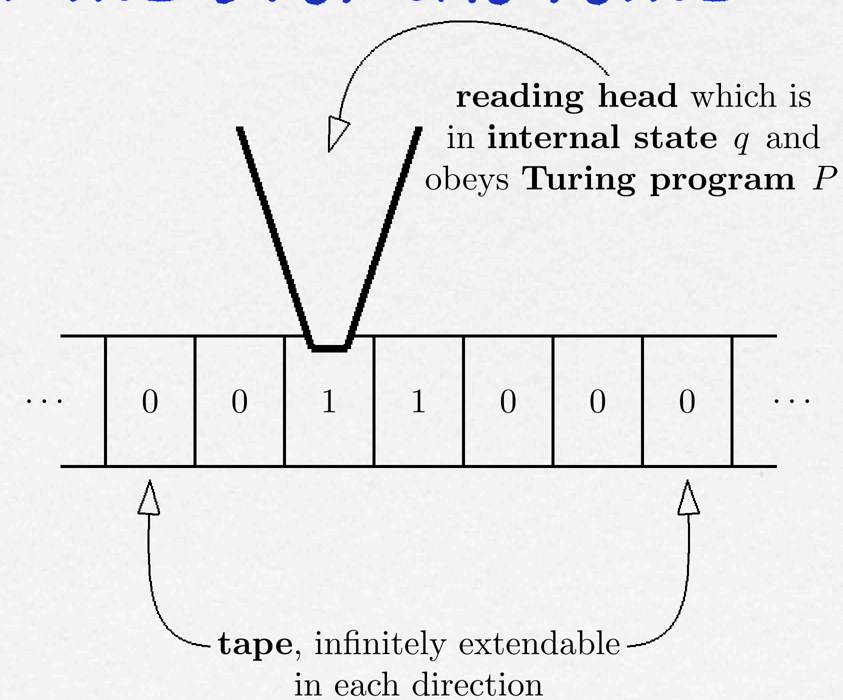
- An explanation of why written proofs do not tell us how the proof was discovered . . .

Towards a basic computational model

- Key ingredients - imaging, parallelism, interconnectivity, and a counterpart to the second-order recursions pointed to above
- Connectionist models - strong on parallelism, interconnectivity, imaging - but not recursions

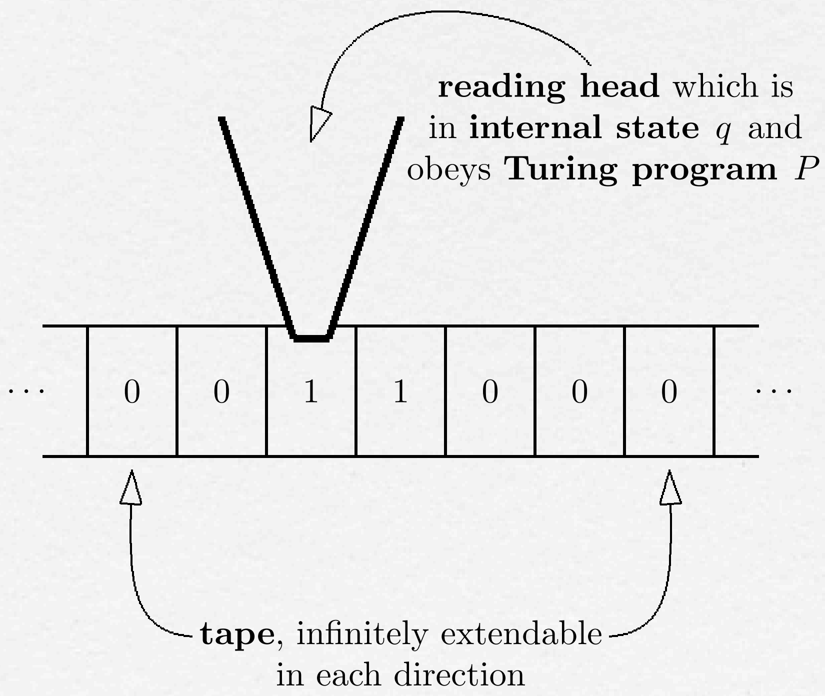
The Turing model extended

- 1939 - Turing's oracle Turing machines appear
- Provides a model of algorithmic content of structures, based on p.c. functionals over the reals



The Turing model extended

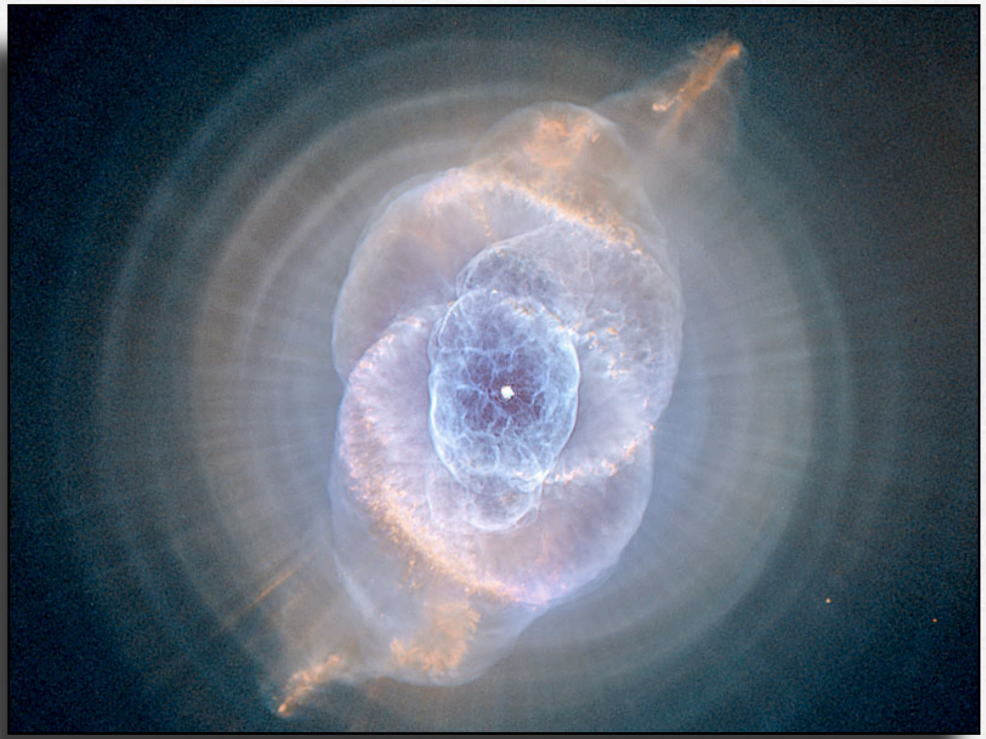
- 1939 - Turing's oracle Turing machines appear
- Provides a model of algorithmic content of structures, based on p.c. functionals over the reals
- A model within which Newton etc comfortably fit ...



The Turing model extended

- 1939 - Turing's oracle Turing machines appear
- Provides a model of algorithmic content of structures, based on p.c. functionals over the reals
- 1944 - Post defines the degrees of unsolvability as a classification of reals in terms of their relative computability
- Giving a landscape with a rich structure

The Turing landscape, causality and emergence ...



- can describe global relations in terms of local structure ...
- ... so capturing the emergence of large-scale formations
- Mathematically - formalise as definability over structure based on Turing functionals?
- More generally - as invariance under automorphisms

Hartley Rogers' programme …

Fundamental problem: Characterise the Turing invariant relations

Hartley Rogers' programme ...

Fundamental problem: Characterise the Turing invariant relations

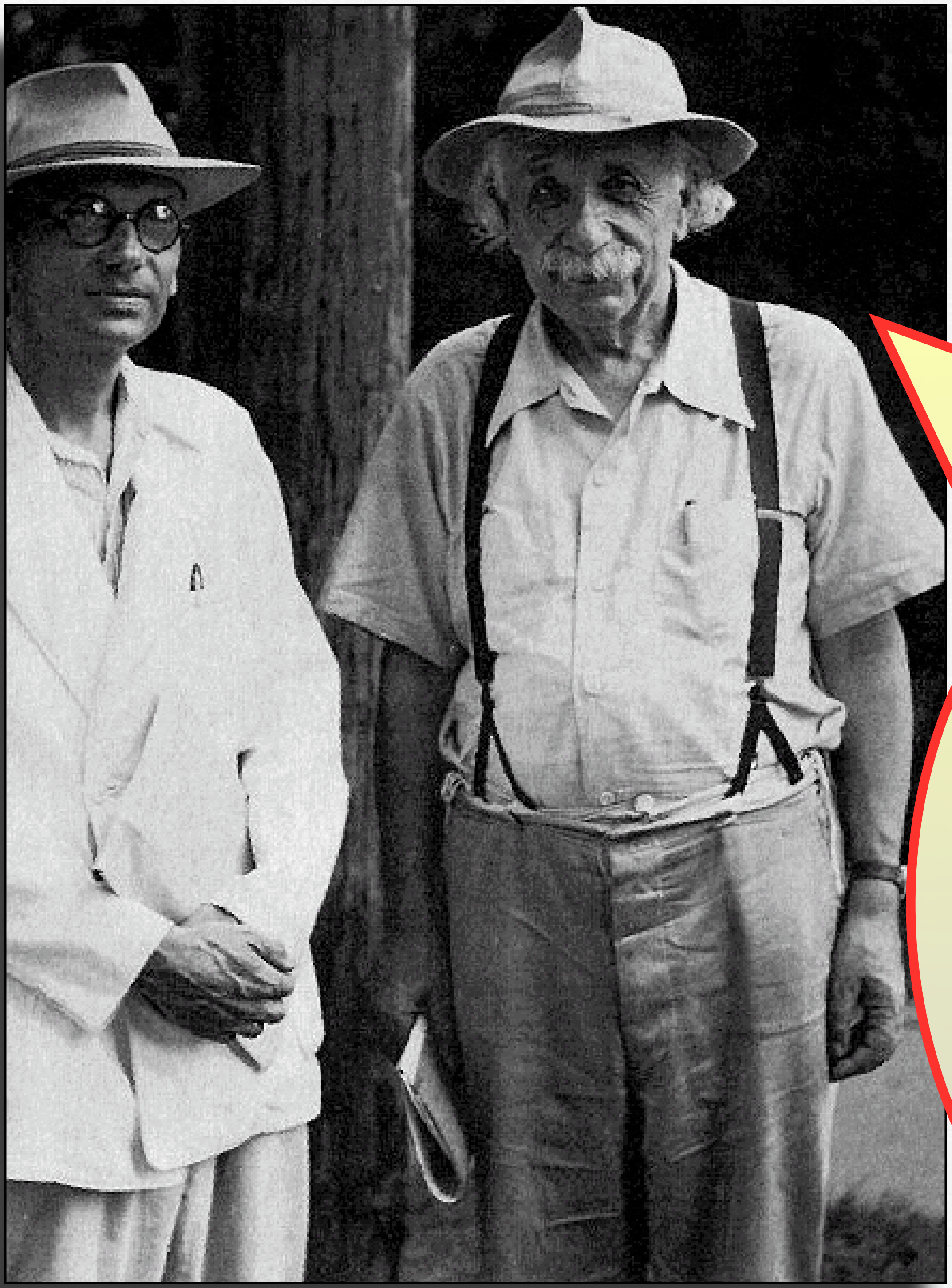
- Intuition: These are key to pinning down how basic laws and entities emerge as mathematical constraints on causal structure
- Notice: The richness of Turing structure discovered so far becomes the raw material for a multitude of non-trivially definable relations

A physics test-drive for the model

By 1973, physicists had in place what was to become a fantastically successful theory of fundamental particles and their interactions, a theory that was soon to acquire the name of the 'standard model'. Since that time, the overwhelming triumph of the standard model has been matched by a similarly overwhelming failure to find any way to make further progress on fundamental questions.

Introduction to Peter Woit: "Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics", Jonathan Cape, 2006

From A. Einstein: "Autobiographical Notes", in "Albert Einstein: Philosopher-Scientist" (P. Schilpp, ed.), Open Court Publishing, 1969, p.63



... I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature ... nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) ...

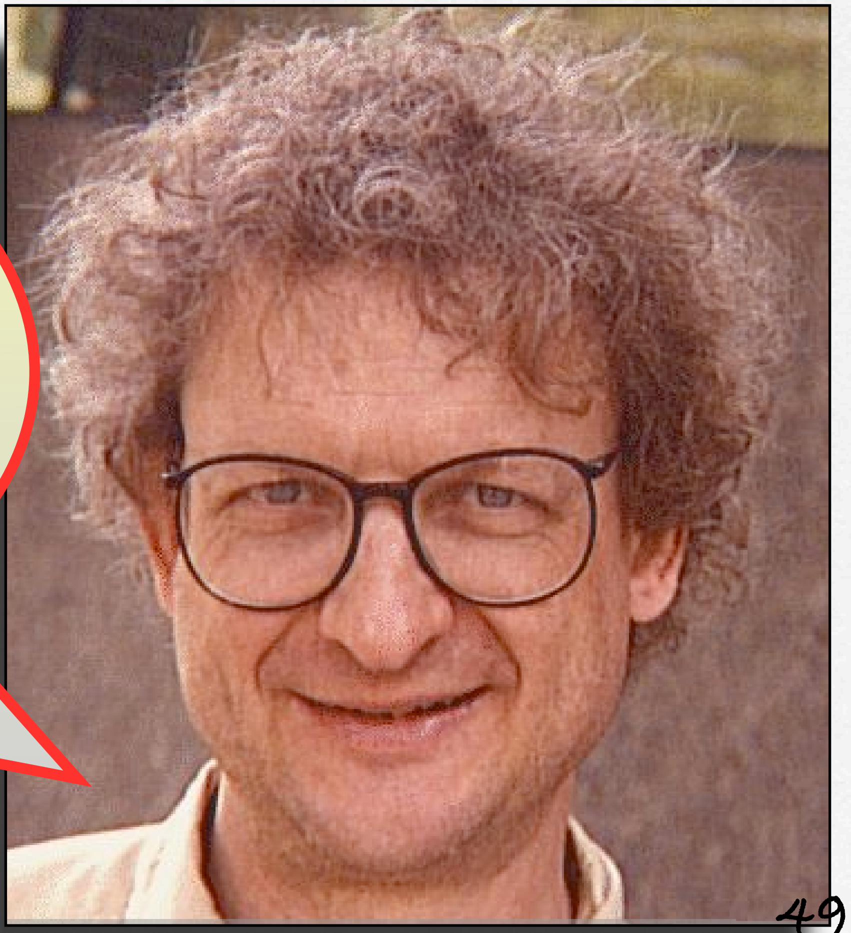
- Peter Woit: "One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ..."

- String theory as a unifying explanatory theory - "the only game in town" ... ?

- Peter Woit: "One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ..."

➡ String theory as a unifying explanatory theory - "the only game in town" ... ?

The longstanding crisis of string theory is its complete failure to explain or predict any large distance physics. ... String theory is incapable of determining the dimension, geometry, particle spectrum and coupling constants of macroscopic spacetime. ... The reliability of string theory cannot be evaluated, much less established. String theory has no credibility as a candidate theory of physics.



Daniel Friedan: A Tentative Theory of Large Distance Physics, J. High Energy Phys. JHEP10(2003)063

Lee Smolin's 5 Great Problems:

1. Combine general relativity and quantum theory into a single theory that can claim to be the complete theory of nature.

2. Resolve the problems in the foundations of quantum mechanics

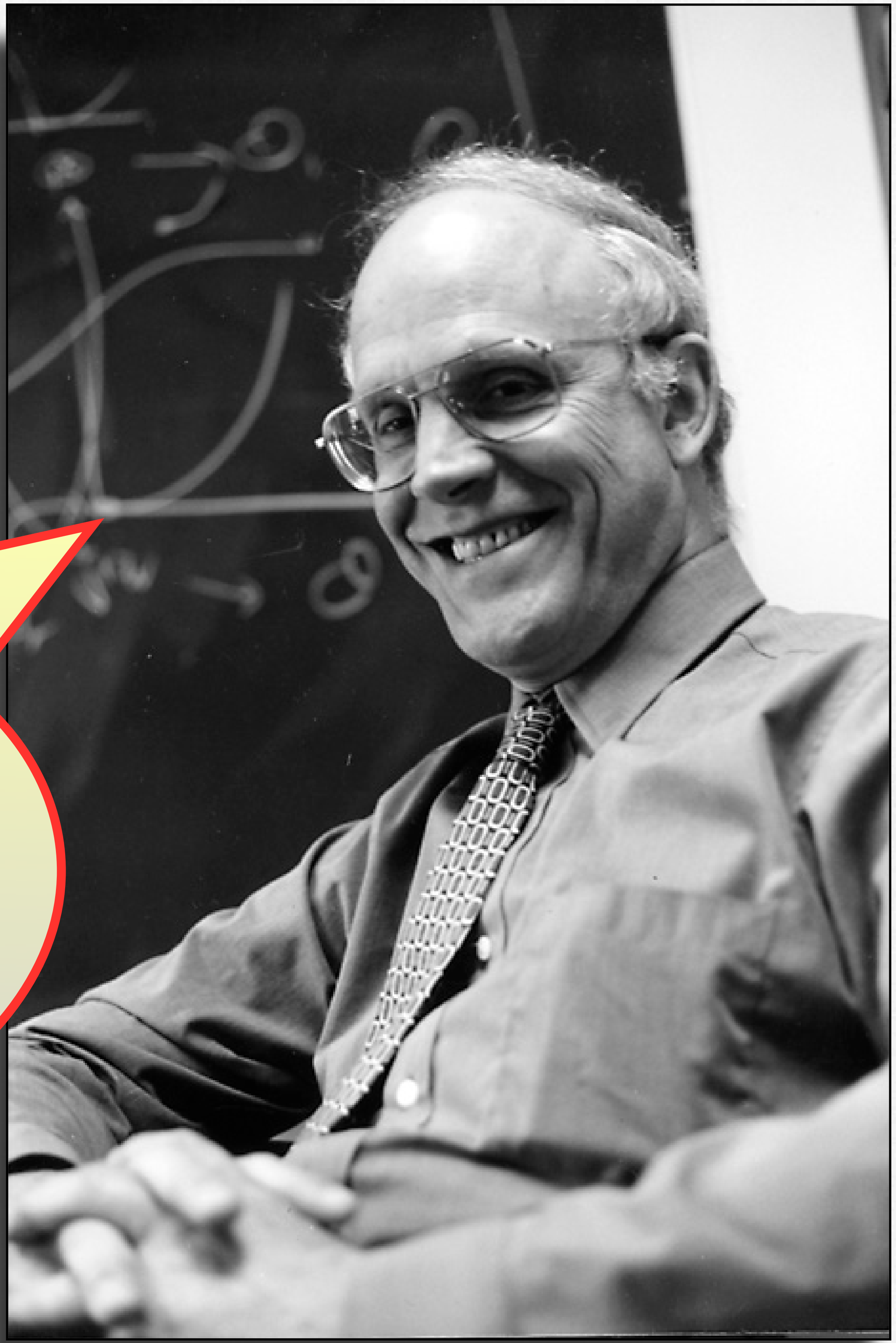
3. The unification of particles and forces problem: Determine whether or not the various particles and forces can be unified in a theory that explains them all as manifestations of a single, fundamental entity.

4. Explain how the values of the free constants in the standard model of physics are chosen in nature.

5. Explain dark matter and dark energy. Or, if they don't exist, determine how and why gravity is modified on large scales.

David Gross, quoted in New Scientist, Dec. 10 2005, "Nobel Laureate Admits String Theory Is In Trouble":

The state of physics today is like it was when we were mystified by radioactivity ... They were missing something absolutely fundamental. We are missing perhaps something as profound as they were back then.



"Causality is fundamental"

- Early champions of the role of causality - Roger Penrose, Rafael Sorkin, Fay Dowker, and Fotini Markopoulou

It is not only the case that the spacetime geometry determines what the causal relations are. This can be turned around: Causal relations can determine determine the spacetime geometry ...
It's easy to talk about space or spacetime emerging from something more fundamental, but those who have tried to develop the idea have found it difficult to realize in practice. ... We now believe they failed because they ignored the role that causality plays in spacetime. These days, many of us working on quantum gravity believe that causality itself is fundamental - and is thus meaningful even at a level where the notion of space has disappeared.

Lee Smolin, The Trouble With Physics, p.241

Quantum uncertainty, Classical emergence

- Emergence Test hard to apply -
- Surprise factor evident! but basic design unclear
- (and part of surprise comes from a failure of emergence)
- Lesson from string theory: When observational base falters, fall back on deep thought and mathematics ...

A deconstructed informational Universe

- Described *in terms of reals* ... With natural laws based on *algorithmic relations between reals*
- Emergence described in terms of *definability/invariance*
- . . . with failures of definable information content modelling *quantum ambiguity*
- . . . which gives rise to *new levels of algorithmic structure*
- . . . and a *fragmented scientific enterprise*

Definability in physical and mathematical contexts ...

Science	Turing landscape
Physical entities modelled as information	Structures information
Theories describing relations over the reals, enabling calculations	Functionals over the reals modelled on human computational capabilities
An extensive basic causal structure which is algorithmic	Models computable causal relations over the reals
Descriptions of globally emerging laws and constants elusive	Problems pinning down the nature of Turing invariance and definability
Quantum ambiguity and nonlocality	Explanation in terms of putative breakdown in Turing definability
Theoretical fragmentation involving phase transitions	Incomputability, and algorithmic relations over emergent objects

Thank you!