The Computational Status of Physics

A Computable Formulation of Quantum Theory

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What this paper is about

I usually argue that hypercomputation is feasible in various models of physics, but it's important to approach boundaries from both sides, so today I'll be trying to do the exact opposite. Yesterday we saw versions of GR that support hypercomputation. But what about other theories? In particular, what about quantum mechanics (QM)?

Strategies

I could try showing that every QM-behaviour can be simulated recursively (by some UQC, say), but if the underlying description of QM is itself uncomputable, I'll be no better off.

Besides, experimental physics is *defined* algorithmically — an experiment is properly specified only if it can be replicated, so you need to define it as an algorithm — so maybe theory is *bound* to say that nature is computable (or is it?) However, this wouldn't be a consequence of *physical law*, but of *social convention*.

Pour-El/Richards to the Rescue

- . . . algorithmically specified systems can generate uncomputable outcomes. . .

- . . . therefore, even though experimental physics is algorithmically specified, the question is still worth asking:

Just how recursive are modern physical theories?

Plan of Action

I'm going to reformulate (one version of) QM in a way that makes it blatantly obvious that QM is behaviourally computational. This doesn't mean hypercomputation is QM-impossible, but it does limit the ways in which QM-hypercomputation might arise.

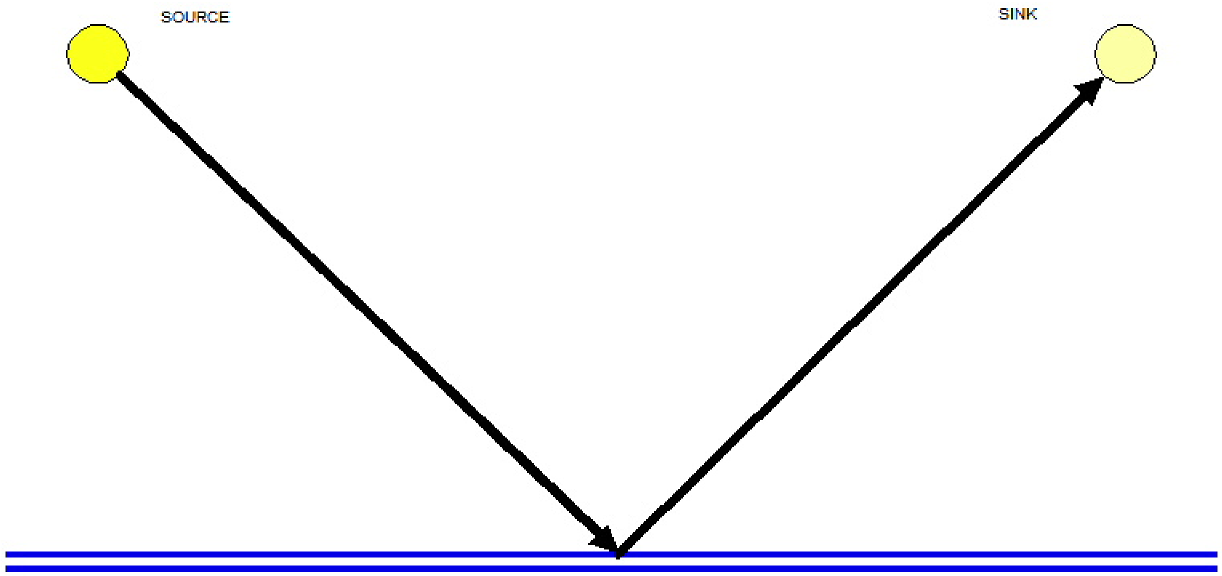
More Generally

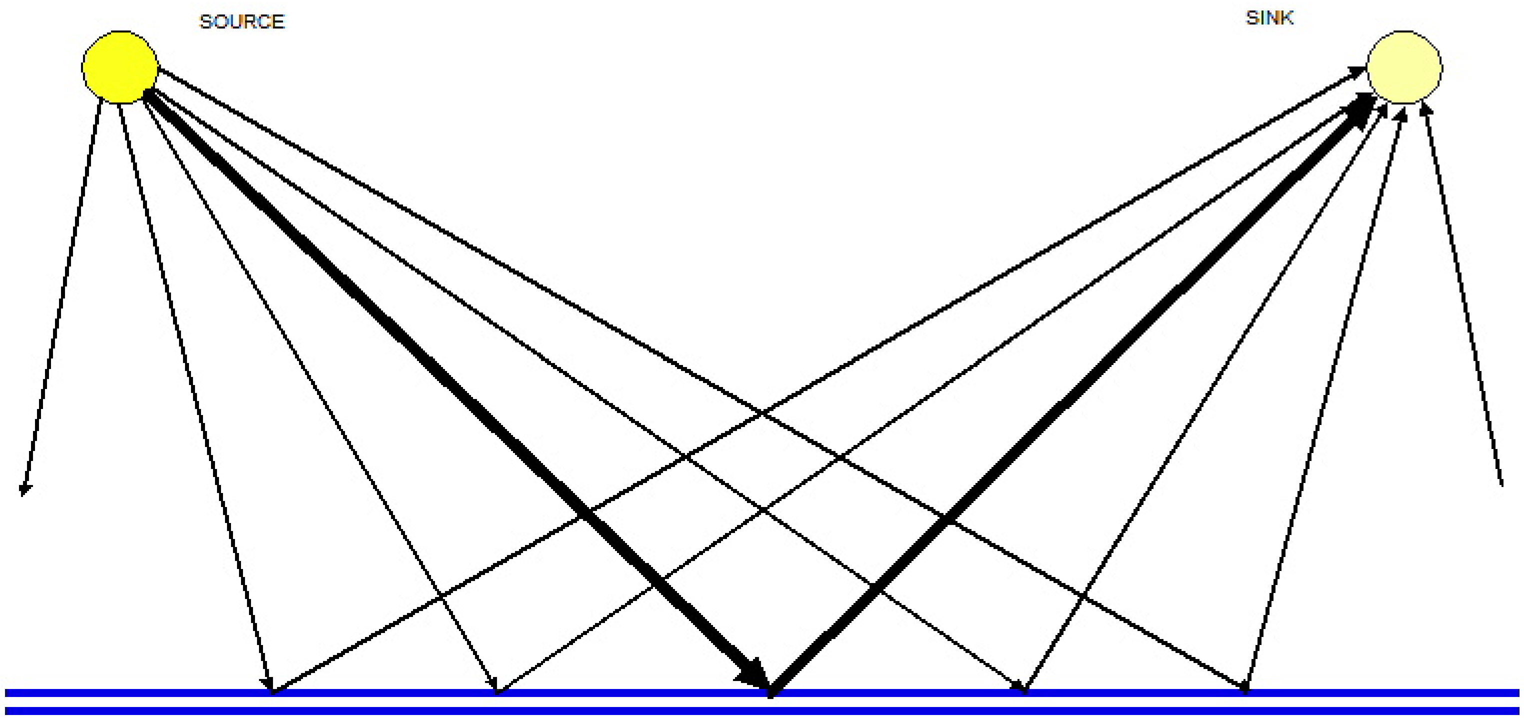
Suppose Φ is any physical theory, expressed in formal logical terms. If every construction used to define physical laws (correct behaviour) in Φ is computable, this could severely limit our ability to construct hypercomputers relative to Φ. Unfortunately it's not always obvious whether Φ is or isn't defined computably.

A construction can *appear* to be uncomputable when it isn't.

Path-Integral Formulation of QM

Famous question - where does light reflect from when you see it in a mirror?

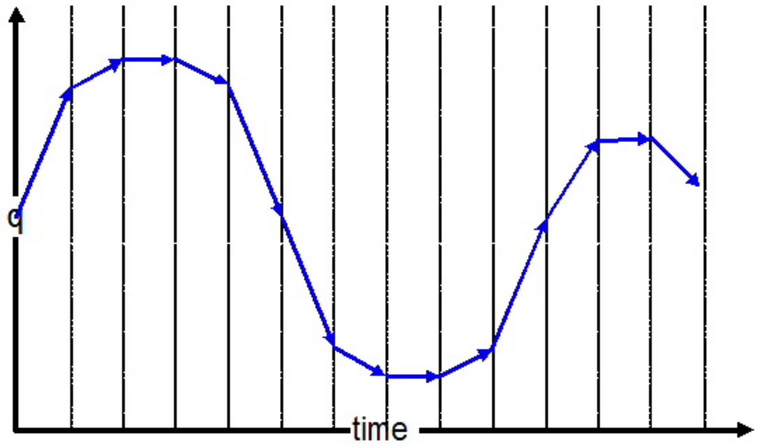




Feynman's answer: All paths are equally likely.

Light reflects everywhere, but *appears* to be reflected in the middle.

Formally. . .



Fix n equally separated times, and guess the particle's position q_m at each of them. Then work out each classical path from q_m to q_{m+1}. Each subpath has a classical *action* S, which you associate with an amplitude $e^{iS/\hbar}$, and their product gives the amplitude for the path. Taking the limit generates one possible path and its amplitude. Now integrate over all possible paths to get the amplitude to go from the start to the finish.

Unjustified Assumptions!

- each trajectory is continuous, so that taking limits is OK;
- trajectory contains infinitely many events;
- particle moving into future (later: piecewise).

Assumptions are common in physics!

Consider e.g. Zuse's model of Newtonian physics using an infinite network of cellular automata (an astoundingly original idea at the time) to generate an early version of *digital physics*.

Flaw: Non-collision singularities in the Newtonian n-body problem.

Point particles can be ejected to infinity in finite time, so by time-reversibility, particles can appear from infinity in finite time. If physics is digital, the spontaneously generated particle must have been deterministically generated by the universe-generating program. But it requires non-local interaction, so can't have been the result of the cellular automata chatting to one another via their local links.

When in doubt, stare at the ceiling!

Why is this a crack?

Interpretation vs Stimuli

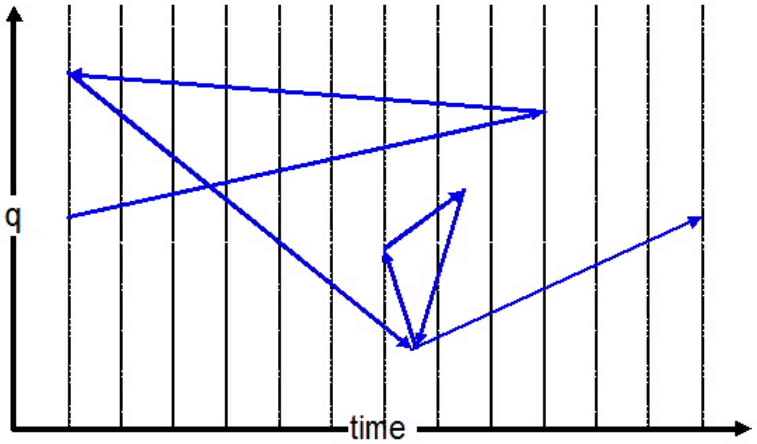
- We like things to appear causal.
- We like time to have a direction.
- We like motion to be continuous-ish.

But we shouldn't simply *assume* such things, because what we see is our brains' interpretation of stimuli; for all we know, causality, time's arrow and continuity are psychological artefacts, not physical ones.

Bidirectional Model

Guess the particle's position and time $x_m = (q_m, t_m)$ at each of n distinct ticks. Then work out each classical path from x_m to x_{m+1}. Each hop has some kind of *hop-action* s_h, which you associate with an amplitude $e^{(is_h/\hbar)}$, and their product gives the amplitude for the path. DON'T TAKE ANY LIMITS; JUST ACCEPT THAT OBSERVATION IS LIMITED. Now sum over all possible paths to get the amplitude to go from the start to the finish.

Trajectory = Finite State Machine



Each possible particle trajectory is an FSM; each such FSM has an amplitude; integrating over all FSMs gives the required amplitude.

Can the equations be made to work?

Theorem: Yes!

Sketch of Proof

In the path-integral model, the amplitude of completing the journey via some journey of n classical sub-paths is some function ϕ_n, and you get the required amplitude by taking the limit, $\phi = \lim \phi_n$.

In our model, we ask ourselves how the motion appears if no observations are made; the composite answer, taking into account all potential observers, is given by some amplitude ψ_0. If we ask how it appears if precisely m observations are made during the relocation from A to B, we get another amplitude ψ_m. Since these possibilities are all mutually exclusive, and account for every possible finitely observed relocation from A to B, the overall amplitude that the relocation happens is the sum of these amplitudes, namely some function $\psi = \sum \psi_m$.

Sketch (continued)

We want $\phi = \psi$ for all possible paths:

$$\lim \phi_n \quad \equiv \quad \sum \psi_m$$

and since

$$\lim_{n\to\infty} \phi_n \equiv \phi_0 + \sum_{n=1}^{\infty} (\phi_n - \phi_{n-1})$$

we can easily solve this requirement by taking

$$\psi_0 = \phi_0 \ ,$$
$$\psi_n = \phi_n - \phi_{n-1} \ .$$

From FSM to X-machines

We can think of particle trajectories as FSMs drawn on spacetime. But this isn't enough to make QM 'computational', because we haven't shown how the amplitudes associated with each FSM are to be 'computed' (as opposed to 'defined'). We do so by thinking of the FSM as an X-machine. This is just an FSM whose labels are relations on some underlying data type X. Technically, I'm using a (simple) variant of the standard XM definition; see the paper.

Trajectories as computations

In this case, we're computing amplitudes, so it makes sense to take $X = \mathbb{C}$. The relation associated with a transition (= hop) of classical action S is then the relation $\lambda z.e^{iS/\hbar} \times z$. With this interpretation,

Each trajectory is a computational state machine that (literally) *computes* its own amplitude

Implications for Hypercomputation

This tells us that QM has computational structure and effect, since all motion can be regarded as computation, and all behaviour involves some sort of 'motion'. But it still doesn't follow that hypercomputation is impossible: the hop-relations $\lambda z.e^{iS/\hbar} \times z$ could still be uncomputable. However, that is (presumably) the *only* way hypercomputation could be achieved in standard QM.

Implications for Physics

The bidirectional model is identically equivalent to the path-integral model; it describes exactly the same behaviours. This means that the assumptions of the path-integral formulation ought to arise as THEOREMS of the bidirectional model. In particular:

- Motion *appears* to be continuous;
- Time *appears* to have an arrow.

Open Questions

- We've ignored GR throughout this construction. What happens if we include it?

- In particular, if you decide to shut off a region of spacetime by saying that anything hopping in can't hop out again, this potentially changes all amplitudes for all bidirectional trajectories (this is NOT obvious in the path-integral formulation). In particular, geodesics will appear to move in the presence of such a 'black hole'. Is this effect already described by GR, or is it new?

THANK YOU!