Black hole computation: implementation with signal machines

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Physics and Computation '08
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Theoretical Physics

Relativization of Church-Turing Thesis

Geometry of space & time $\rightsquigarrow$ Limits of computation

In particular

Some Black hole geometries allow to go beyond classical limits...

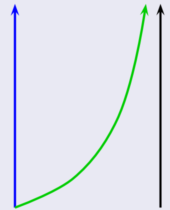
... by using different world lines with incommensurable time scales

- they have a common point
- the entire future of one is in the casual past of one point in the other (after a finite length/(local-)duration)

Possible settings

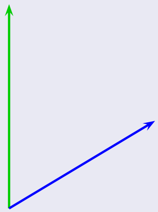
The **observer** starts the **machine**
and sets it on the “faster” world line

Observer infinitely slowed down



Duration for the observer to reach the asymptote is *finite*

Machine infinitely accelerated



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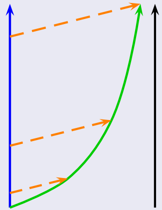

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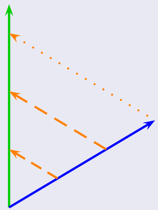

Time available to the machine is *infinite*

The machine can send an **atomic piece of information**
The observer would get it within a known finite duration

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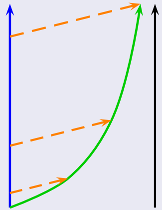
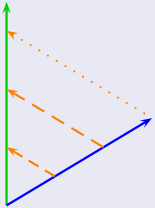


Duration for the observer to reach the asymptote is *finite*

Observer stuck in the singularity

Machine infinitely accelerated



Time available to the machine is *infinite*

Observer can use the singularity again and again

The machine can send an **atomic piece of information**
The observer would get it within a known finite duration

Abstract level

- *One* iteration for the observer
- *Unboundedly many* iterations for the machine

Relates to

- infinite time Turing machines
- computing with ordinal time

But

- only an atomic piece of information can be sent
- it should be sent after finitely many iterations
- no “limit operator”

Computing power

At some point, the observer knows that anything that would have ever been sent would have been received

If the machine sends a signal only if it stops. . .

the halting problem can be decided *by the observer*!

The first level of the arithmetical hierarchy can be decided

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Our aim

to translate it into *Abstract geometrical computation*

An idealization of

- Collision computing
- Signals in cellular automata

Signals, particles, solitons, filtrons...

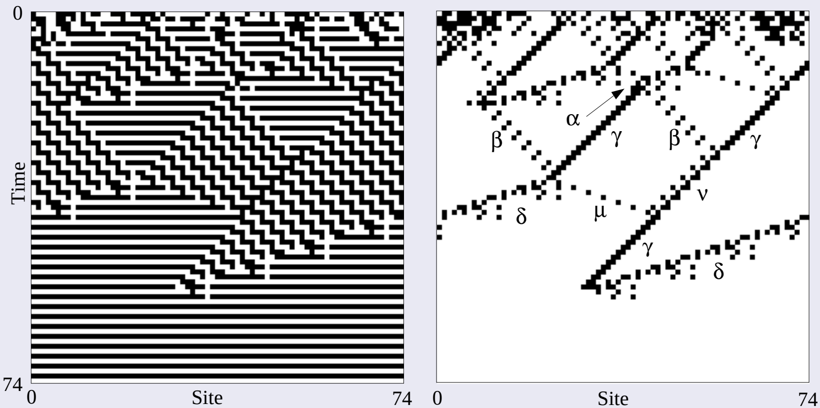
- Information conveyors
- Uniform movement

Collisions, encounters...

- Update informations (and carriers)
- Only available interaction

$\rightsquigarrow$ perform computations without wires nor gates

Signals in CA context



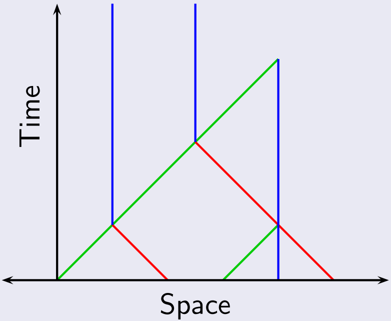
(a) Space-time diagram. (b) Filtered space-time diagram.

Abstract geometrical computation

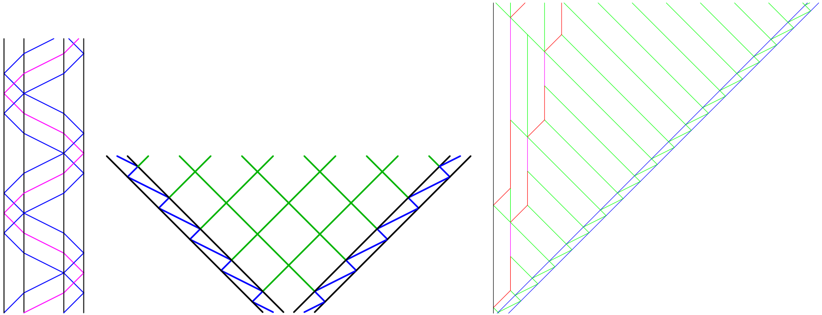
Idealization

- Signals are dimensionless
- Uniform propagation depending only on the information carried
- Finitely many signals (finite description)

Space-time diagram



Examples



AGC Primitives

"Meta-programming"

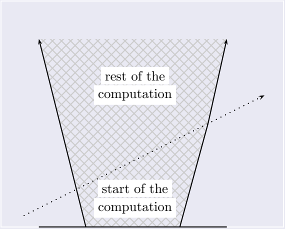
Signal machines and configurations are built from existing ones by adding signals and rules

Various constructions for various effects

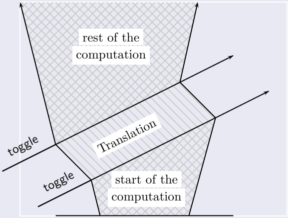
Original computation "preserved"

Freezing and unfreezing

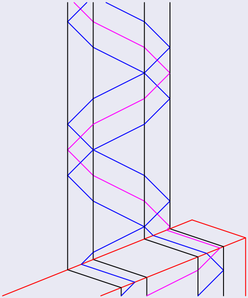
Regular evolution



Frozen, translated and unfrozen

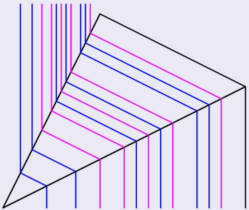


Example



Scaling/contracting

Principe on parallel signals



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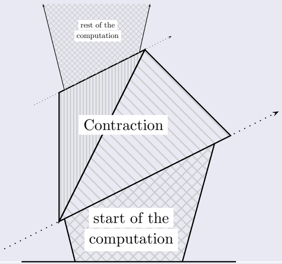
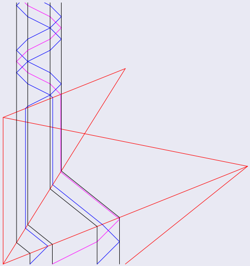
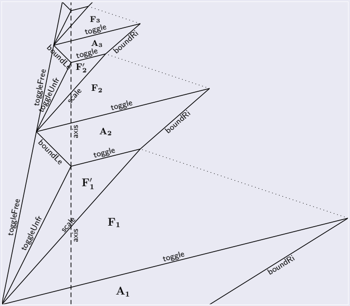


Example



(Infinite) Shrinking...

Iterated scaling



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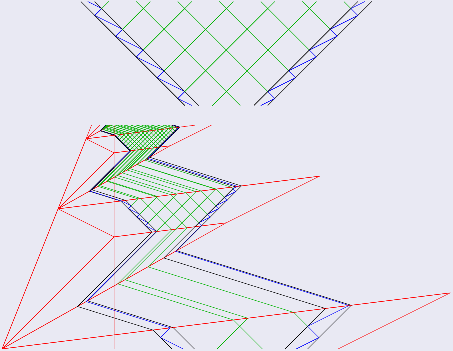

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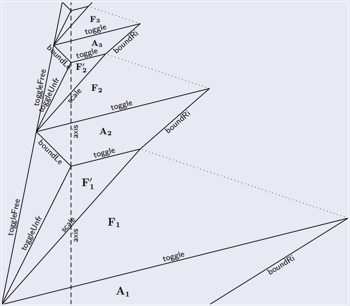

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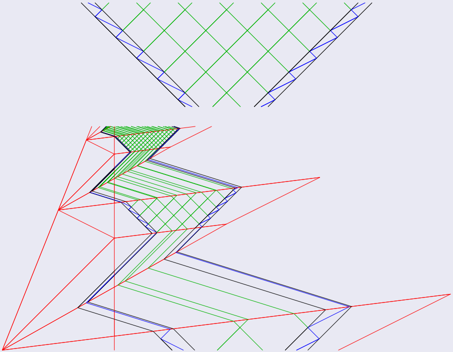
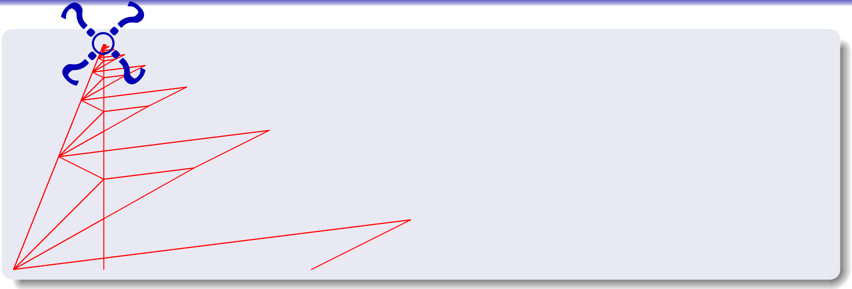


Converging geometrical progression

Any computation starting from a bounded configuration can be folded into a finite part of the plane

...generates an accumulation



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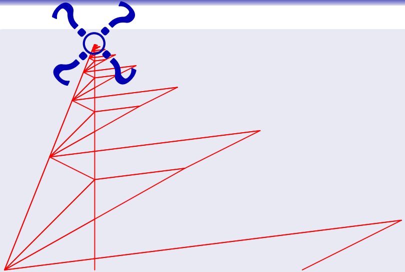
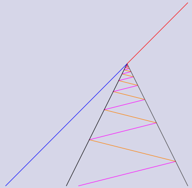


Dealing with accumulations



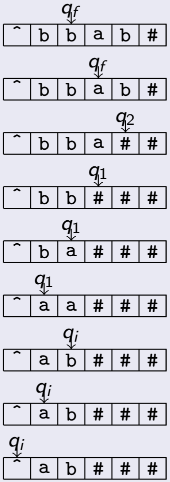
Various schemes can be defined

For this talk

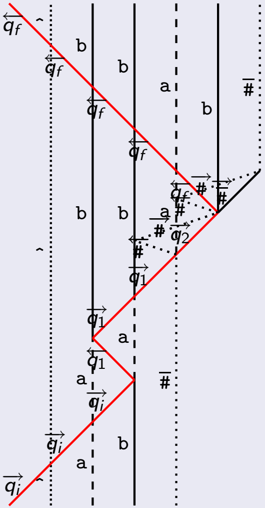
Signals are emitted according to *incident signal(s)*

Computing (in the usual understanding)

Modeled by Turing machines

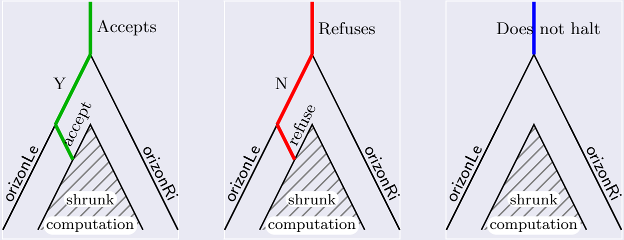


Simulation



Shrinking the computation

- Whole (potentially infinite) computation embedded inside a finite part of the space-time diagram
- Problem: recovering any "result"
  - add meta-signals and rules to let some information leave
  - add signals out-side to collect it



Computing power

Let R be any recursive total predicate
(a computable total function from $\mathbb{N}$ to $\{\texttt{YES}, \texttt{NO}\}$)

Shrink a machine that tests R for every value and
stops on the first YES (and sends a signal)

The observer

can decide any formula of the form

$$\exists n \in \mathbb{N},\ \text{recursive total predicate on } n$$

This is the first level of the *arithmetical hierarchy* Σ_0^1

- halting problem
- conjecture decision
- consistency of logics

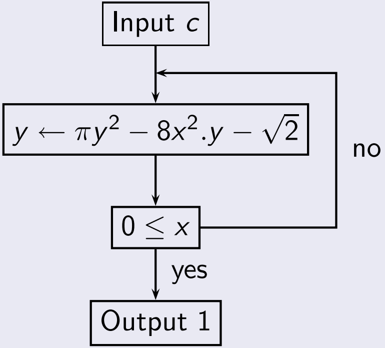
Comparing to the black hole model

- Shrinking provides the infinite acceleration
- An atomic piece of information can leave
- Same computing power

Model considered

There is no Turing thesis $\rightsquigarrow$ a lot of incomparable models

Blum, Shub and Smale model $(\mathbb{R}, +, -, *)$



D-L CiE '07

AGC *without accumulations*
is equivalent to
BSS without inner multiplication

D-L CiE '08

AGC *with accumulations*
can simulate the full BSS

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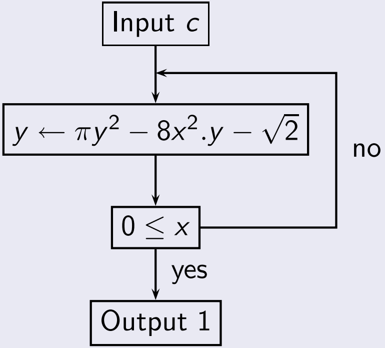

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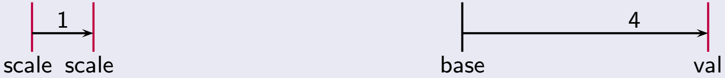
Question

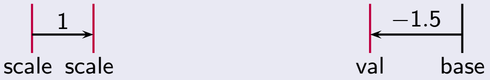
How does this behave with accumulations/black-holes?

Encoding



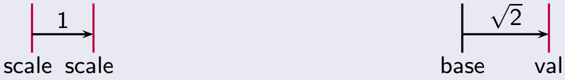
Value: 4

Encoding



Value: -1.5

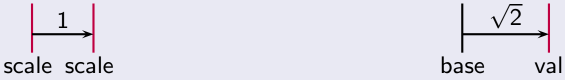
Encoding



Value: $\sqrt{2}$

- As far as all the computation is folded, there is no problem
- An atomic piece of information may leave

Encoding



Value: $\sqrt{2}$

- As far as all the computation is folded, there is no problem
- An atomic piece of information may leave

Limits

A real number is encoded by four signals

- they might leave at the accumulation at different foldings
- at the accumulation all signals meet

Computing power

So that only an atomic piece of information, *i.e.* a *digital* one

BSS-arithmetic hierarchy

It corresponds to $\exists n \in \mathbb{N}$ over a BSS total predicate

It is not an analytical hierarchy since

- all integers can be listed
- but not all real numbers

Multiplication already uses accumulation

How does it work?

- 3 signals are fixed
- The accumulation takes place where the 4th one should be and is generated according to the accumulation scheme

Interaction with the shrinking structure

- Structure away from accumulation... OK
- Structure on the accumulation...
  its signals are incident distinguishing the accumulation... OK

Infinitely many multiplications...

$\rightsquigarrow$ second order accumulation!

Folding on folding

Iterated construction

- Each construction generates new signals and rules
- The scheme used distinguishes the cases

The level has to be defined from start and cannot be changed

Effect

Climbing the levels of the corresponding arithmetic hierarchies (*i.e.* alternation of $\exists$-$\forall$ quantifiers on $\mathbb{N}$)

It correspond to SAD_n
(arithmetical sentence deciding space-times) of Hogarth

Results

Similarities

- sub-computation has infinite time to run
- it can send an atomic piece of information (after a finite time)
- same computing power

Main difference

- singularities are generated (*vs.* they have to be found)

Thank you for you attention