P Systems with Mobile Membranes : A Survey

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Overview

- Introduction
- Mobile Membranes : A variant of active membranes
- Results

P Systems with Active Membranes

- Introduced by Gh. Păun (Nov 1998)
- Polarized membranes with charges +, -, 0
- Division of elementary membranes based on opposite polarization, no label change
- Very powerful : solves hard problems, universal.

P Systems with Active Membranes

- Polarized Membranes : Remote inspiration from biology
- Price to pay for removing polarizations : division of non-elementary membranes, change labels of membranes, use cooperative rules.
- Improvement : Two polarizations suffice for universality (Alhazov, Freund, Păun)
- Can we remove polarizations and still have the power?

Mobile Membranes

- Price to pay in removing polarizations : introducing new operations on membranes
  - Endocytosis, Exocytosis : movement of elementary membranes in/out of membranes
  - Elementary membrane division without polarizations

P Systems with Mobile Membranes

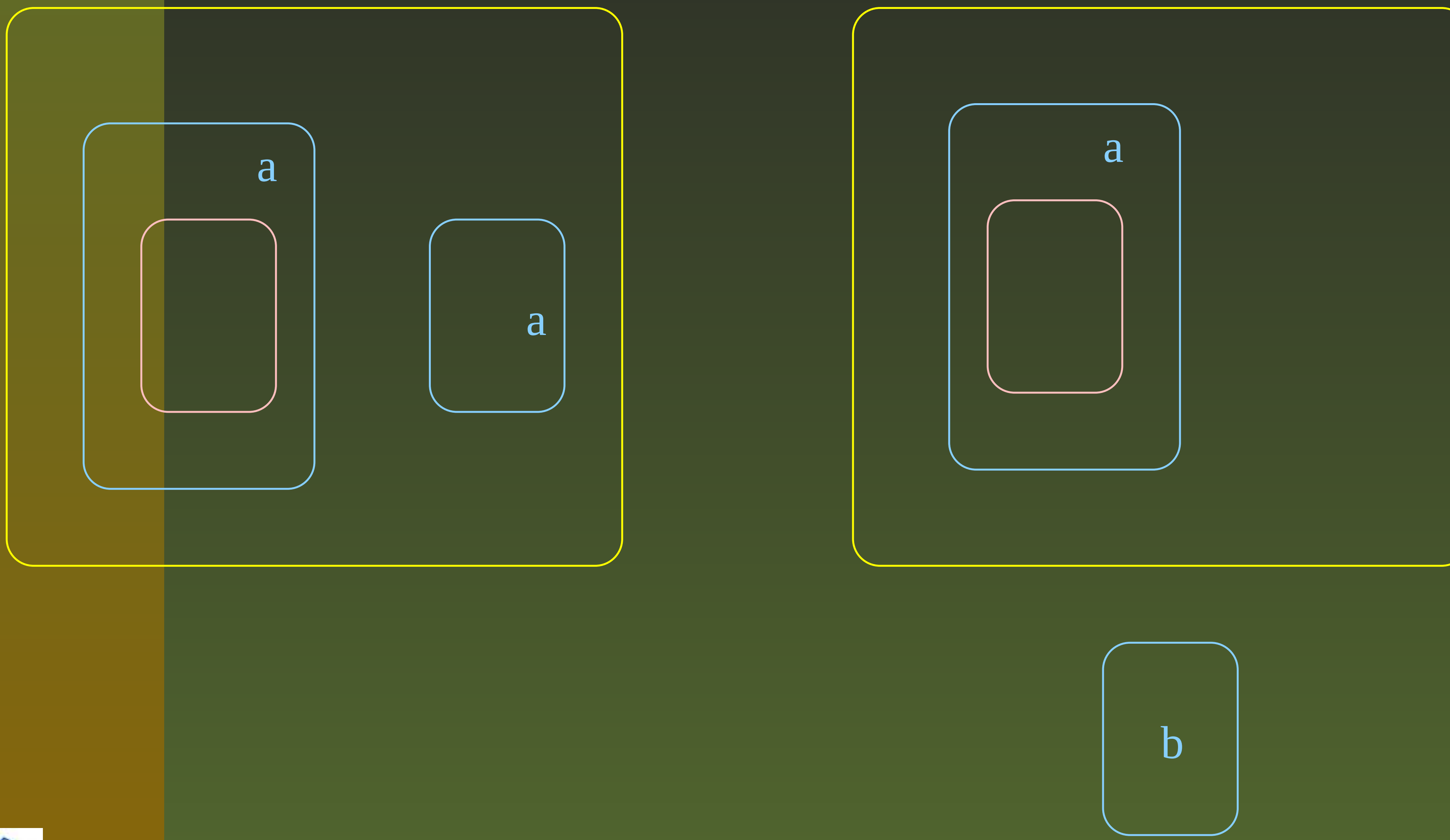
- $\Pi = (V, H, \mu, w_1, \ldots, w_n, R)$
  - V is the basic alphabet of *objects*,
  - H is a finite set of *labels* for membranes,
  - μ is the membrane structure,
  - w_i are strings over V in regions i, and
  - R is a finite set of rules.

Types of Rules

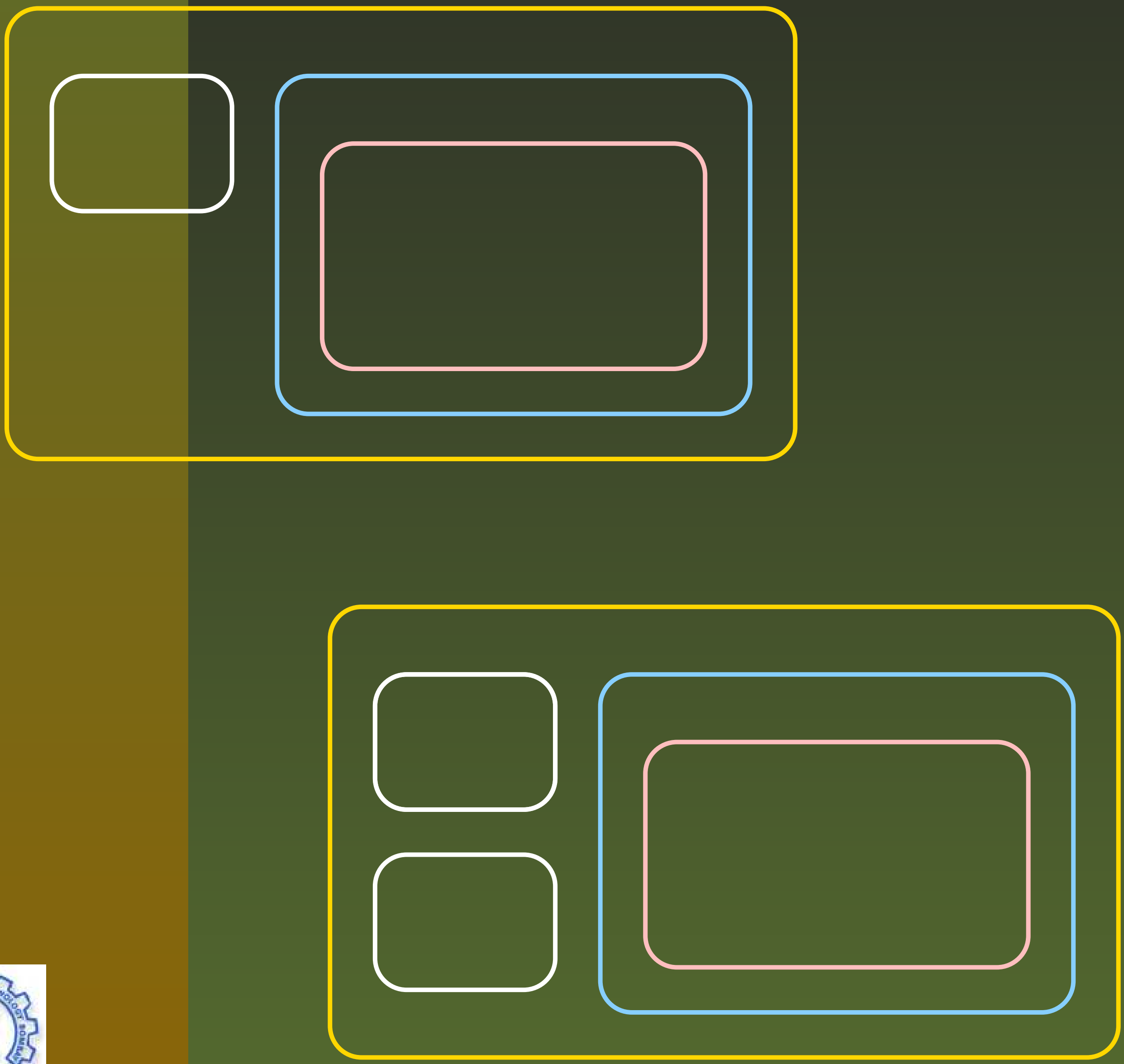
- Endocytosis : For $h, m \in H$, h elementary, $a, b \in V$,
  - $[_h a]_h [_m \;]_m \rightarrow [_m [_h b]_h]_m$
- Exocytosis : For $h, m \in H$, h elementary, $a, b \in V$,
  - $[_m [_h a]_h]_m \rightarrow [_h b]_h [_m \;]_m$
- Object Evolution : For $m \in H, a \in V, v \in V^*$,
  - $[_m a \rightarrow v]_m$
- Elementary Division : For $h \in H, a, b, c \in V$,
  - $[_h a]_h \rightarrow [_h b]_h [_h c]_h$

Endocytosis

Exocytosis



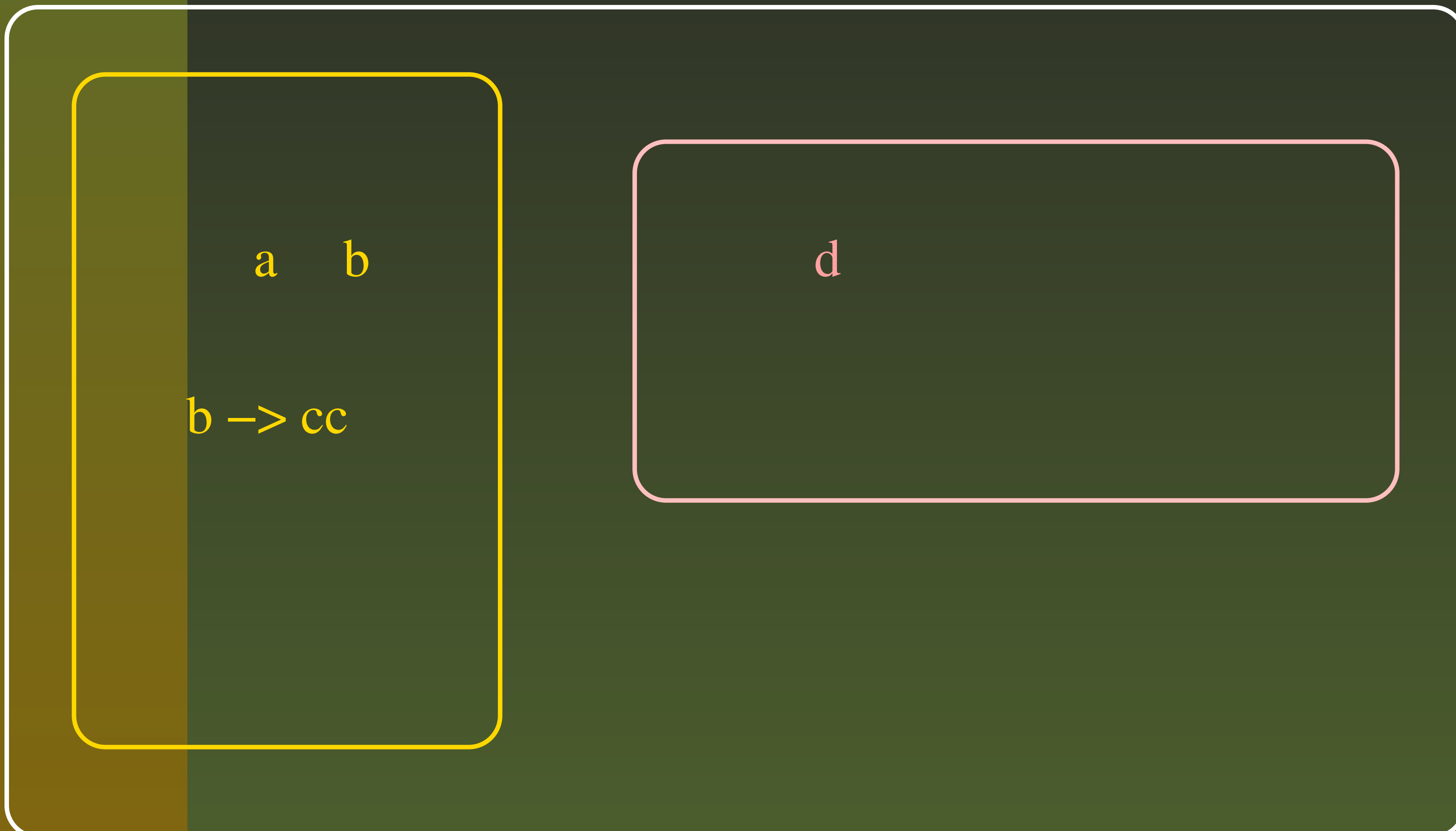
Elementary Membrane Division



Output and Language Generated

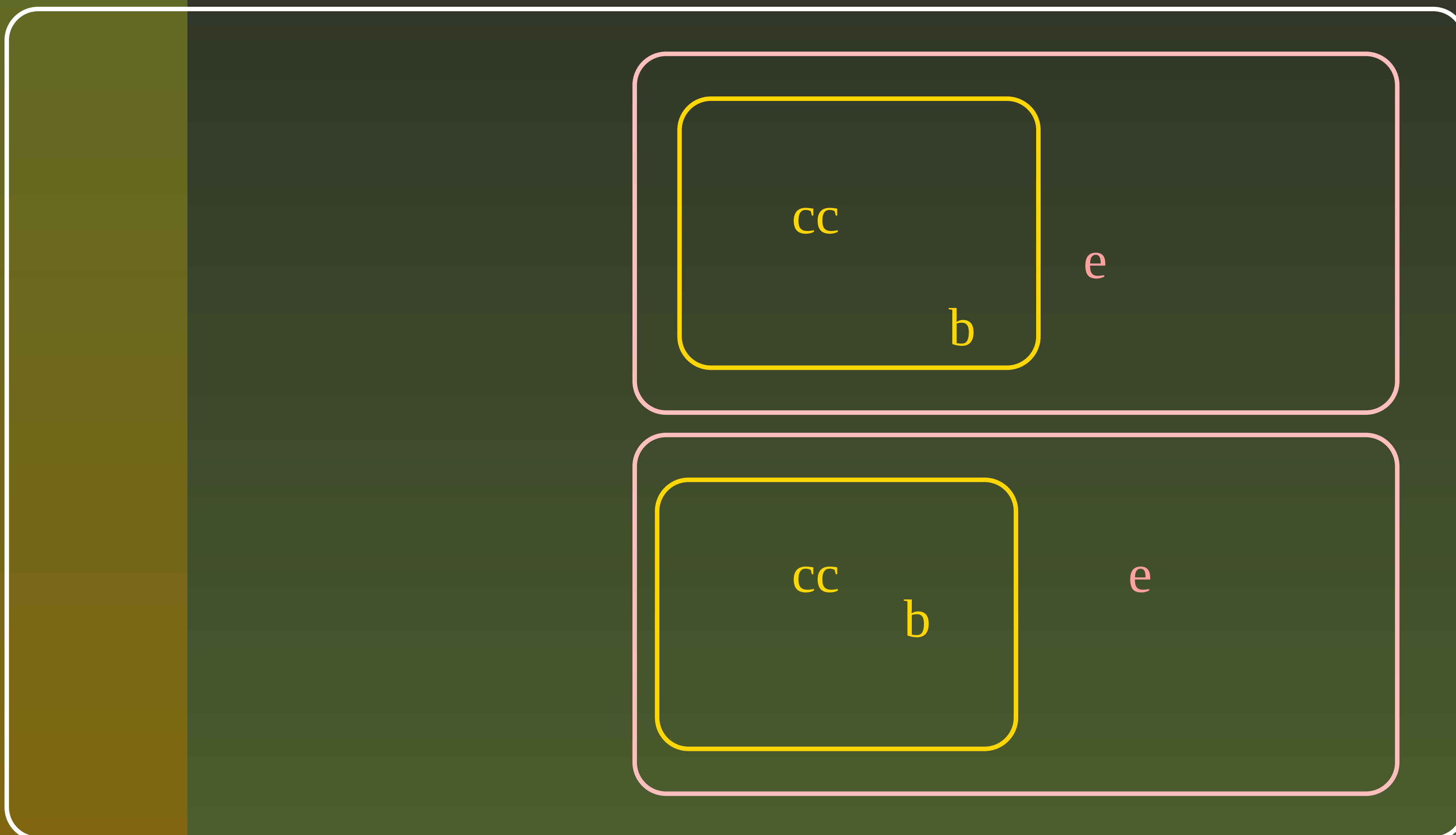
- At the end of a halting configuration, observe all membranes sent out of Π by exocytosis.
- All vectors describing multiplicity of all objects from all such membranes forms the output vector.
- The set of all output vectors of Π is denoted by $Ps(\Pi)$.
- The family of all sets $Ps(\Pi)$ generated by systems Π of degree $\leq n$, using the operations $endo, exo$ is denoted by $PsMP_n(endo, exo)$.

An Example



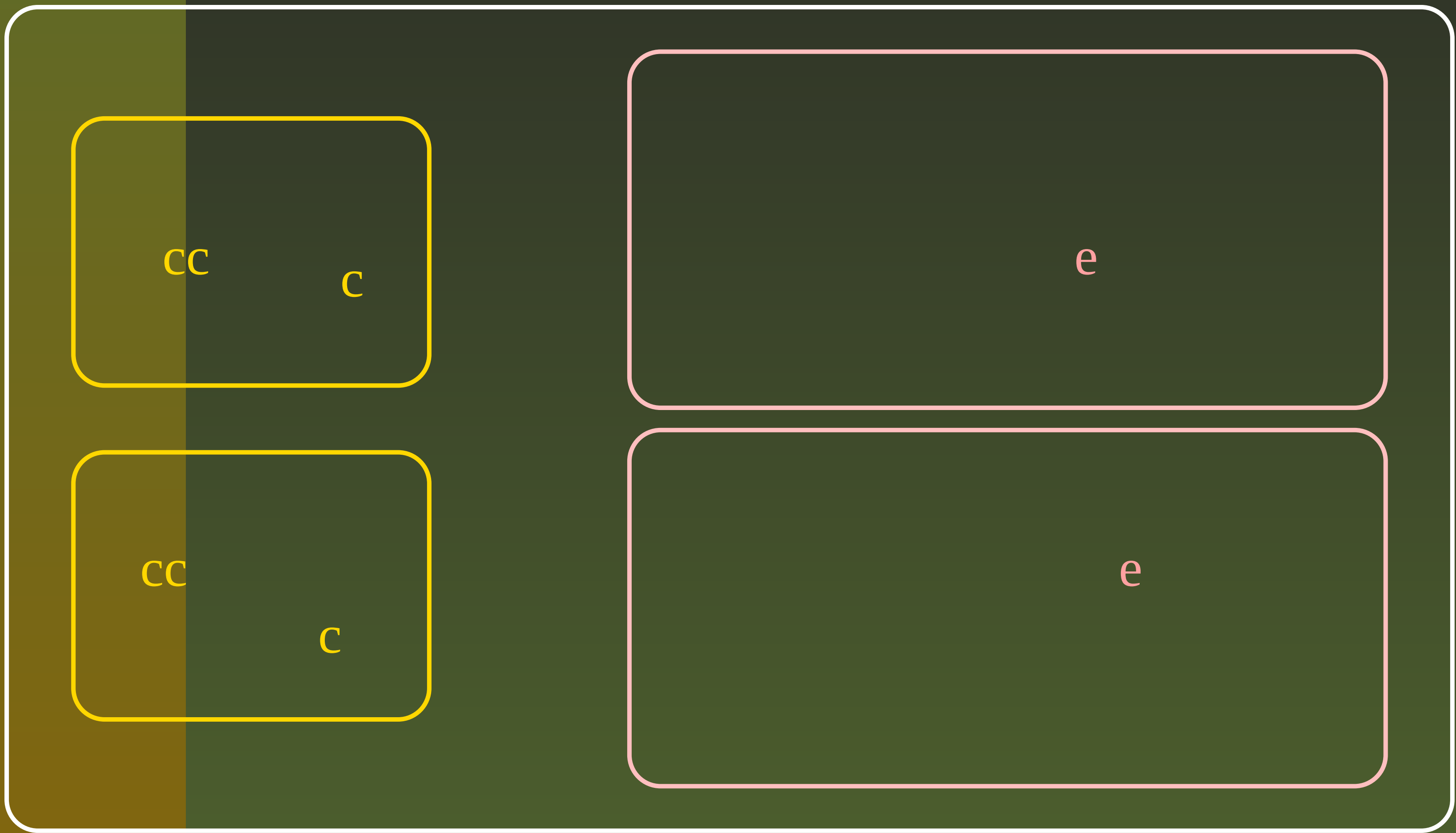
[a] [ ] → [[b]] [d] → [e] [e]

An Example



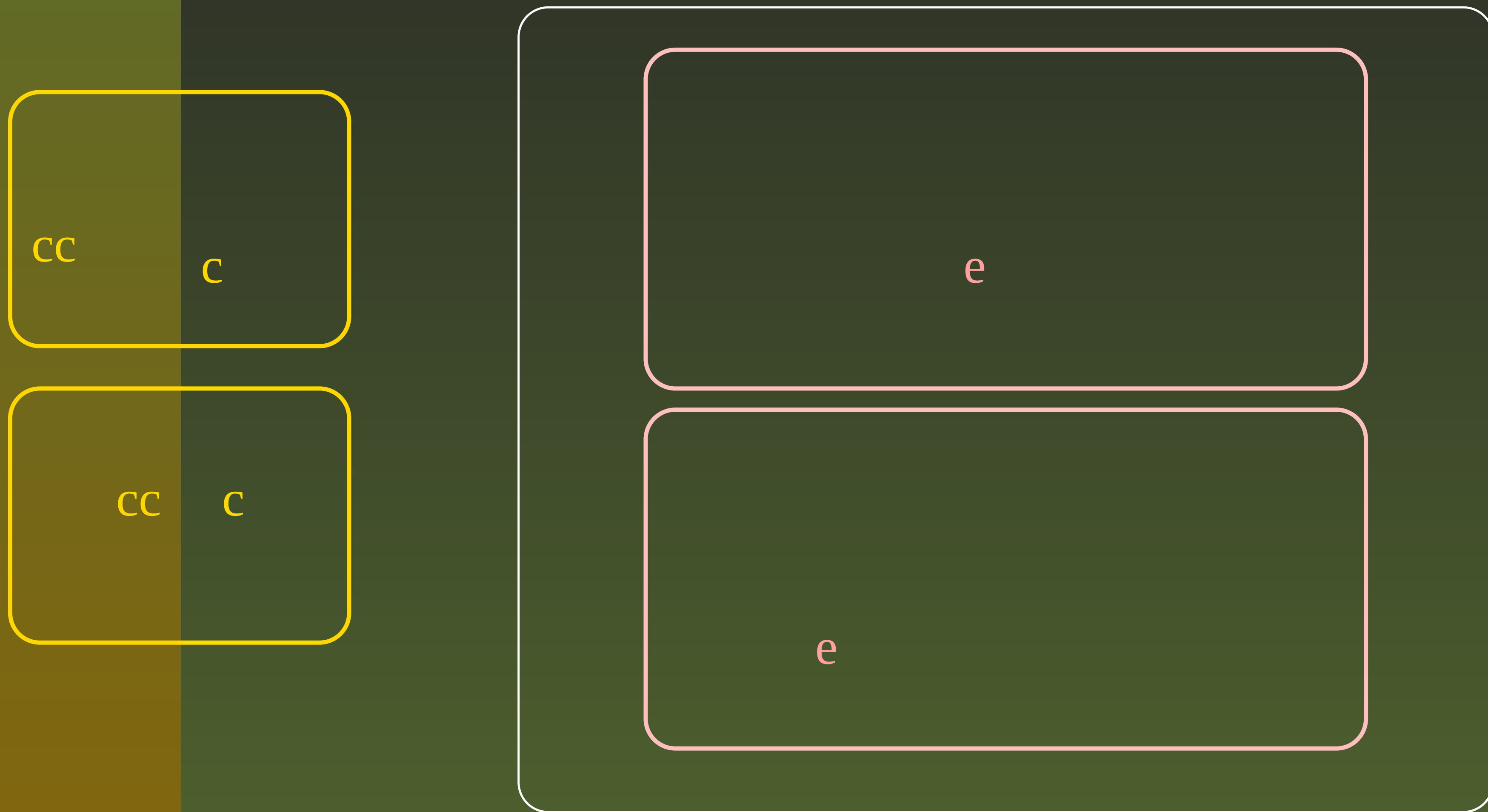
[[b]] ⟶ [ ] [c]

An Example



[[c]] ⟶ [c] [ ]

An Example



An Example

- We had $V = \{a, b, c, d, e\}$

- $Ps(\Pi) = \{(0, 0, 3, 0, 0), (0, 0, 3, 0, 0)\}$

Universality

- Simulating a known variant of P systems
  - P systems with replicated rewriting
- Direct universality without using division
  - $PsMP_9(endo, exo) = PsRE.$

Universality

- Replicated Rewriting - string objects
- Mobile Membranes - symbol objects
- Simulation of a system with symbol objects using a system with string objects

P Systems Replicated Rewriting

- P Systems with Replicated Rewriting
  $\Pi = (V, \mu, M_1, \ldots, M_n, R_1, \ldots, R_n)$ with
  - $M_1, \ldots, M_n$ are finite languages over V,
  - R_i rules of the form
    $a \rightarrow (u_1, tar_1) || \ldots || (u_k, tar_k)$, $k \geq 1$,
    $a \in V, u_i \in V^*$ and $tar_i \in \{in, here, out\}$
  - $x_1 a x_2$ transformed into $x_1 u_1 x_2$ in $tar_1, \ldots$
    $x_1 u_k x_2$ in tar_k.

P Systems Replicated Rewriting

- Parikh images of set of strings sent out of the system $Ps(\Pi)$ at the end of a halting computation : output
- $PsSP_n(repl_d)$: Family of all sets $Ps(\Pi)$ computed by systems with $\leq n$ membranes and using d replication.
- $PsSP_3(repl_2) = PsRE$.

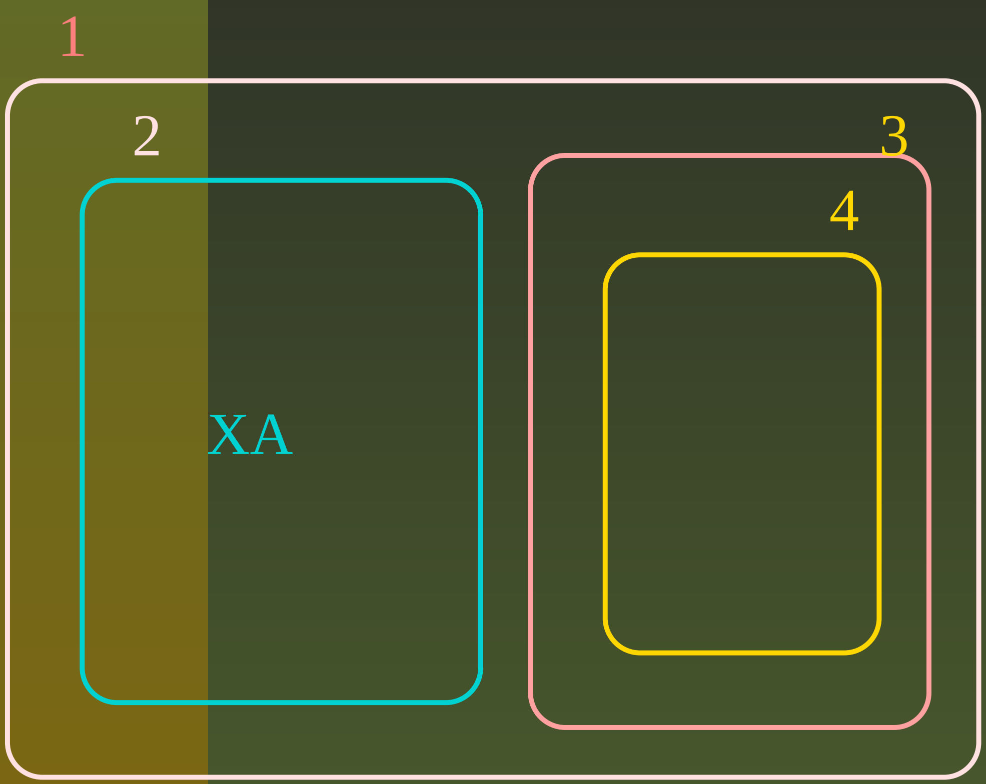
Simulation of Replicated P Systems using Mobile Membranes

- Every replicated P system Π with 2 replication can be simulated by a P system with mobile membranes Π' with local division such that each transition in Π is simulated in Π' in at most 3 steps, such that $Ps(\Pi) = Ps(\Pi')$.
- Replicated rewriting P systems with 2 replication are universal. This gives the universality of P systems with mobile membranes.

Improved Universality

- $PsMP_4(endo, exo) = PsRE$
- Universality obtained by simulation of a matrix grammar $G = (N, T, S, M, F)$ in *improved strong binary normal form*.
  - $N = N_1 \cup N_2 \cup \{S, \#\}$ with matrices of the form
  - $(S \rightarrow XA)$, with $X \in N_1, A \in N_2$,
  - $(X \rightarrow Y, A \rightarrow x)$, with $X, Y \in N_1, A \in N_2, x \in (N_2 \cup T)^*, |x| \leq 2$,
  - $(X \rightarrow Y, B^{(j)} \rightarrow \#)$, with $X, Y \in N_1, j = 1, 2$,
  - $(X \rightarrow \lambda, A \rightarrow x), X \in N_1, A \in N_2, x \in T^*$.

Improved Universality



Improved Universality

- Simulation of a type 2 matrix $(X \to Y, A \to x)$
  - A symbol $A \in N_2$ is replaced by an indexed symbol using an $endo$ rule making an entry into membrane 3.
  - X is rewritten in parallel in membrane 2 into an indexed symbol
  - The indices of both symbols are checked for equality, using $endo/exo$ rules of membranes 2,4.
  - If unequal, membrane 2 gets blocked inside membrane 3 or 4.

Improved Universality

- Simulation of type 3 matrices $(X \to Y, B^{(j)} \to \#)$
  - An *endo* rule is made from membrane 2 to 3, replacing X by an intermediate symbol;
  - If the corresponding symbol of N_2 $(B^{(j)})$ is present, an *endo* rule is used, entering membrane 4; the intermediate symbol evolves in parallel in membrane 2;
  - If membrane 2 is adjacent to membrane 4, then $B^{(j)}$ is absent

Further Results

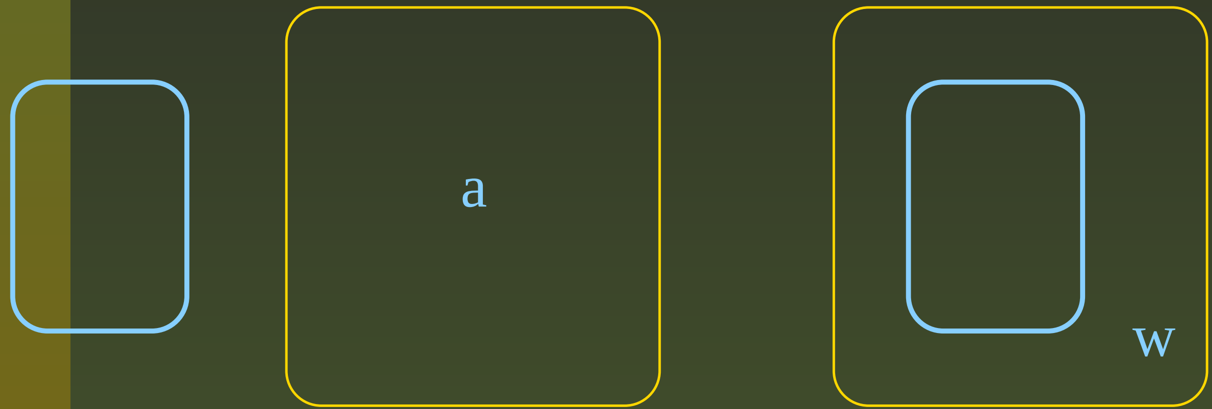
- $PsMP_2(exo) - PsMAT \neq \emptyset$
- $PsMP_2(exo) \subseteq PsRC_{p,f} \subset PsRE$
- $Ps0L \subseteq PsMP_2^{\lambda}(exo)$
- $PsMP_3(endo, exo) = PsRE.$
  - Best known universality result - proved by simulating 2 counter machines
  - Open : How expressive is $PsMP_2(endo, exo)$?

P Systems with Enhanced Mobile Membranes

- $\Pi = (V, H, \mu, w_1, \ldots, w_n, R, i)$, with rules of the kind
  - $[a \to v]_m$ (object evolution)
  - $[a]_h[\,]_m \to [[w]_h]_m$ (endocytosis)
  - $[[a]_h]_m \to [w]_h[\,]_m$ (exocytosis)
  - $[\,]_h[a]_m \to [[\,]_h w]_m$ (forced endocytosis)
  - $[a[\,]_h]_m \to [\,]_h[w]_m$ (forced exocytosis)
  - $[[a]_j[b]_h]_k \to [[w]_j[b]_h]_k$ (contextual evolution)
  - $[a]_h \to [b]_h[c]_h$ (elementary membrane division)

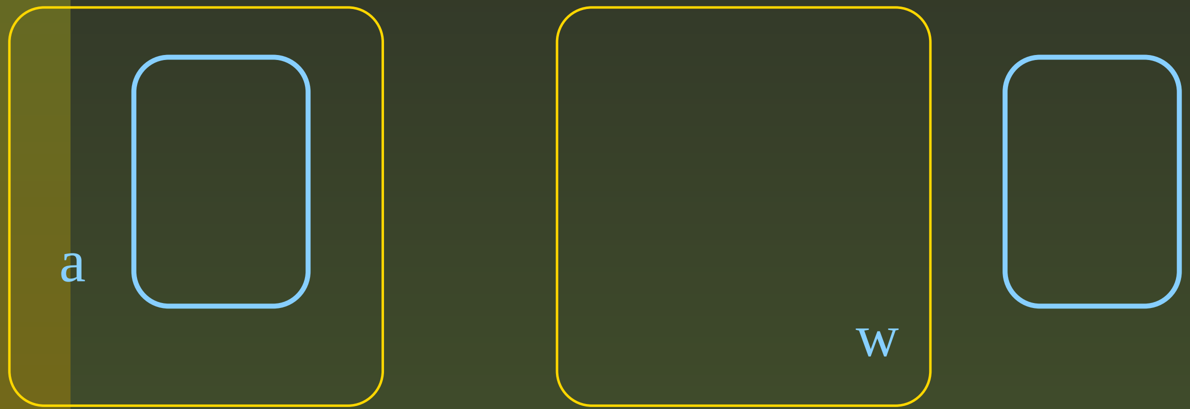
P Systems with Enhanced Mobile Membranes

- Forced Endocytosis



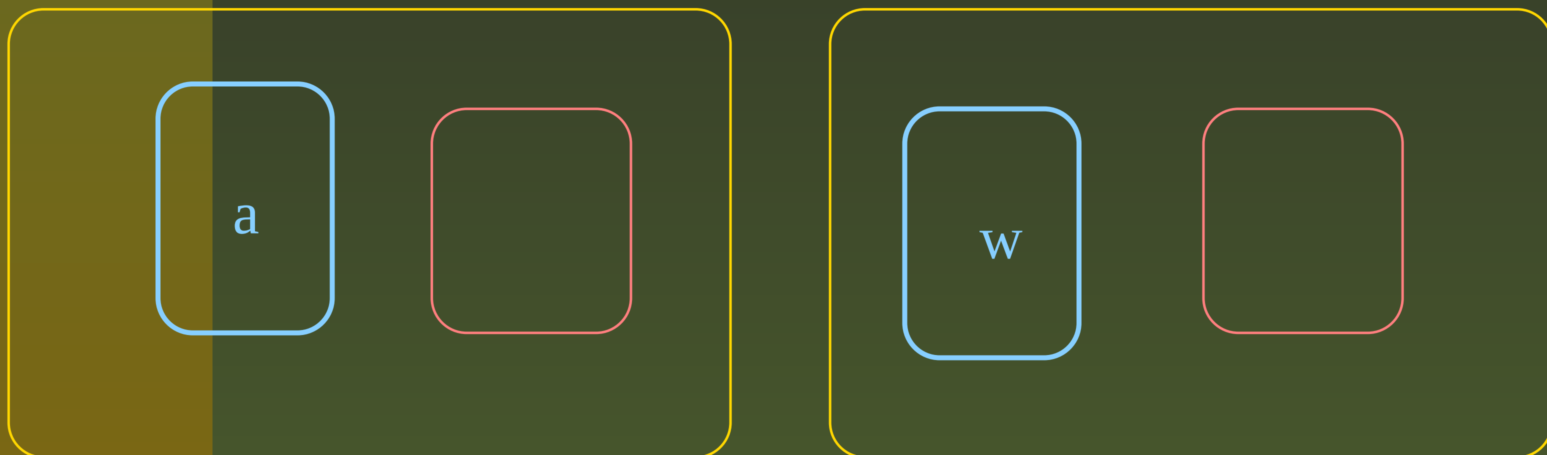
P Systems with Enhanced Mobile Membranes

- Forced Exocytosis



P Systems with Enhanced Mobile Membranes

- Contextual Evolution



P Systems with Enhanced Mobile Membranes

- At the end of a halting configuration, observe the output membrane. Vectors describing the multiplicity of objects in that membrane form the output.
- The family of all sets $Ps(\Pi)$ generated by systems of degree $\leq n$, using operations $\alpha \subseteq \{endo, exo, fendo, fexo, cevol\}$ is denoted $PsEM_n(\alpha)$.

P Systems with Enhanced Mobile Membranes

- $PsEM_{12}(endo, exo, fendo, fexo) = PsRE$
- $PsEM_3(cevol) = PsRE$
- $PsE0L \subseteq PsEM_7(endo, exo, fendo, fexo)$
- $LEM_8(endo, exo, fendo, fexo) - ET0L \neq \emptyset$
- $PsEM_3(endo, exo) = PsEM_3(fendo, fexo)$
- $PsEM_3(endo, exo, fendo, fexo) \subseteq PsMAT$

P Systems with Enhanced Mobile Membranes

- Applications : Modelling the immune system, Mobile Ambients (G. Ciobanu, B. Aman)
- Questions
  - further applications
  - what is a useful and decidable sub class of enhanced mobile membranes
  - optimality of theoretical results obtained